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A Railroad Track Structural Analysis Method for Work Planning: Development and Example Application

by
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This report documents the development of a railroad track structural analysis method, describes an example application, and includes an executable computer program on diskette. This method was originally developed for the Railroad Engineered Management System (RAILER).

The method is intended to help provide an estimate of an existing track's suitability to handle its expected loading and to permit an assessment of the potential effects of changes in the track--either improvements or deterioration. It employs five equations that provide values for rail bending stress, tie bending stress, tie reaction force, ballast surface stress, and subgrade surface stress. The equations are designed for conventional track and will handle the full range of track conditions and loads normally encountered.

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FOREWORD

This research was conducted for the U.S. Army Engineering and Housing Support Center (USAEHSC) under Project 4A162731AT41, "Military Facilities Engineering Technology"; Task C, "Operation, Management, and Repair"; Work Unit 042, "Railroad Maintenance Management System (RAILER)." The work was performed by the Engineering and Materials Division (EM), U. S. Army Construction Engineering Research Laboratory (USACERL). The USAEHSC Technical Monitor was Robert Williams, CEHSC-FB-P. His support is very much appreciated.

The Principal Investigator for the RAILER project is D.R. Uzarski. The authors wish to thank him for the independence he granted them in developing the track structural analysis equations and for his faith in the successful outcome of this effort.

A special note of appreciation is extended to David Coleman of the U.S. Army Waterways Experiment Station for his assistance in the early phases of this work.

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Dr. Paul A. Howdyshell is Acting Chief, USACERL-EM.

COL Everett R. Thomas is Commander and Director of USACERL, and Dr. L.R. Shaffer is Technical Director.

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1 INTRODUCTION

Background

Much of the Army's 3000-mile railroad network was built during World War II using secondhand track materials and expedient construction methods. The track, even by contemporary standards, was often not well suited for heavy loads.

Since their construction, railroads on Army installations have been required to handle increasingly heavier cars and wheel loads. These loads are commonly well beyond the original design loads for the track. In addition, track maintenance has been quite variable over the years, often leaving the track's load-carrying ability in question.

With the Army's railroad system playing an important role in mobilization plans and training exercises, there was concern about the ability of the track to handle the expected loading. The requirement for carrying heavier loads, combined with the generally light construction and variable maintenance, created the need for a method by which installation Directorates of Engineering and Housing (DEHs) could determine track suitability for mission loading.

To assist the DEH in improving the management of general track maintenance, the U.S. Army Engineering and Housing Support Center (USAEHSC) asked the U.S. Army Construction Engineering Research Laboratory (USACERL) to develop an Engineered Management System (EMS). The resulting product is the Railroad Engineered Management System (RAILER).[1]

Within RAILER, there had to be a way to assess the track's ability to handle its intended load: a structural analysis procedure. This procedure would indicate the need to perform maintenance or rehabilitation to correct structural deficiencies in the track.

The RAILER track structural analysis procedure, in turn, needed a track analysis method that was simple enough to be consistent with the rest of the RAILER system, yet have a moderately high degree of versatility and accuracy. Existing track analysis tools tended toward one extreme or the other, with none encompassing all the required characteristics. Thus, a new method was needed.

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[1] M.Y. Shahin, *Development of the U.S. Army Railroad Track Maintenance Management System (RAILER)*, Technical Report M 86/01/ADA168915 (U.S. Army Construction Engineering Research Laboratory [USACERL], May 1986); D. Uzarski, D. Plotkin, and D. Brown, *Maintenance Management of U.S. Army Railroad Networks--The RAILER System: Component Identification and Inventory Procedures*, Technical Report M 88/13/ADA200276 (USACERL, August 1988); D. Piland and D. Uzarski, *The RAILER System for Maintenance Management of U.S. Army Railroad Networks: RAILER I Computer User's Guide*, ADP Report M-88/16/ADA199611 (USACERL, September 1988); D. Uzarski, D. Plotkin, and D. Brown, *The RAILER System for Maintenance Management of U.S. Army Railroad Networks: RAILER I Description and Use*, Technical Report M-88/18/ADA199859 (USACERL, September 1988).

Objective

The objective of this work was to develop a railroad track structural analysis method that meets the following requirements:

1. Is simple to use
2. Provides enough information to allow a basic track structural evaluation to be made
3. Has a reasonably high degree of accuracy
4. Is capable of handling the full range of track conditions and loads normally encountered
5. Can be handled easily using a microcomputer.

Approach

After examining information on existing structural analysis tools, a large-scale track structural analysis computer program was chosen as a reference for creating the simplified method. Five key parameters were then selected to form the output from the method.

From this point, the intent was to form five single, and relatively simple, mathematical expressions or equations that would yield relatively accurate values for the five chosen output parameters. Because the use of layered elastic and finite element methods in the program precluded easy extraction of single or simple equations, the approach was to attempt to create these equations from observed behavior of the five output parameters.

The next step was to study the behavior of these five parameters using a wide variety of track characteristics. The most important of these (appearing to have the greatest influence on the five output parameters) were then selected to become variables in the equations.

The final equations were developed by combining studies of the output from several thousand runs of the reference computer program and output from conventional analytical methods with an acquired sense of parameter behavior.

Mode of Technology Transfer

It is expected that this track structural analysis method will be incorporated into the RAILER System, which is being transferred to the field. In addition, application of this method can be taught through presentations and training courses on railroad engineering and maintenance practices.

This method will be included in a revision of Technical Manual (TM) 5-850-2, *Railroad Design and Construction*, for use in design planning and evaluating existing track.

2 TRACK STRUCTURAL ANALYSIS

Requirements for the Track Structural Analysis

The purpose of a structural analysis is to determine if the track structure is suitable to handle the loads it is expected to carry. This structural analysis can be performed at varying levels of detail and with a variety of methods.

Existing track structural analysis methods tend toward two extremes. On one end of the scale are quick analytical tools such as the "40 percent rule" for determining tie reaction and pressure distribution diagrams for determining vertical stress in the ballast and subgrade.[2] These methods are simple to use, but their accuracy is limited because they do not take into account different material properties or the important interaction between the track components.

On the other end of the scale are the complex track structural analysis computer programs. These tools do allow for different material properties and consider the interactions among track components. However, they also require special knowledge to run and to interpret the output, and they demand a large amount of computer space.

What was desired for the RAILER EMS was a structural analysis between these two extremes. Ideally, it would have the simplicity of the quick analysis tools with the accuracy and versatility of the large computer programs.

General Approach in Developing the Method

Since the large track structural analysis computer programs had the advantage of accuracy and versatility, it seemed logical to look for a procedure that could approximate their results, with the output then simplified to the desired degree. After examining several programs, the KENTRACK model, developed at the University of Kentucky, was selected as a basis for comparison.[3] Thus, the project goal was now defined as creating a procedure that would approximate the results produced by the KENTRACK program.

For simplicity, it was desired to have a small number of expressions, or equations, to evaluate the four basic track system components: rail, ties, ballast, and subgrade (Figure 1). The approach would be to create equations representing basic analytical parameters for each component--generally, maximum bending or surface stresses. These equations would be based on those variables most influential in predicting each parameter.

The next step was to investigate the behavior of the track-related variables within the KENTRACK program and select those having significant effect on each analytical parameter.

The final, and most challenging, step was attempting to create relatively simple mathematical expressions that would closely approximate the behavior produced by the track structural analysis program.

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[2]W.W. Hay, *Railroad Engineering*, 2nd ed. (John Wiley & Sons, 1982).

[3]Y.H. Huang et al., *KENTRACK, A Finite Element Computer Program for the Analysis of Railroad Tracks, User's Manual* (Department of Civil Engineering, University of Kentucky, February 1986).

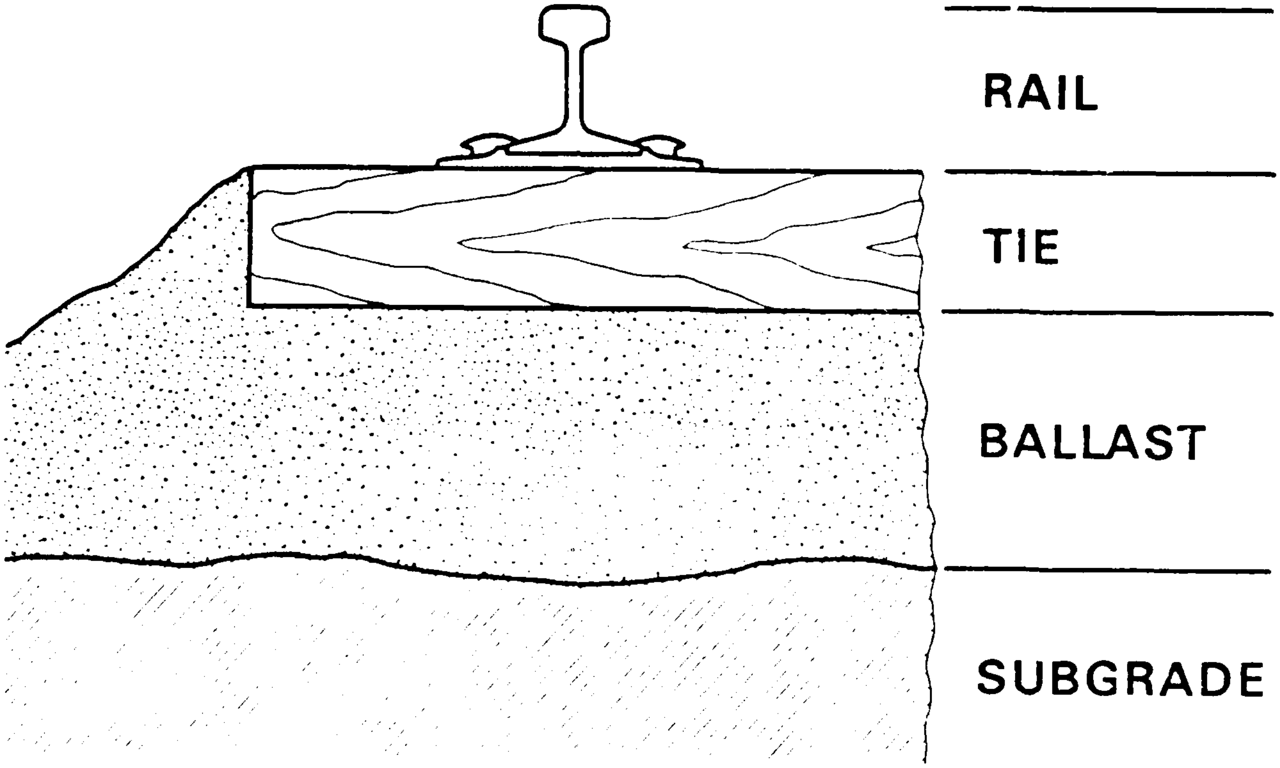

Figure 1. The four major track system components.

3 DEVELOPMENT OF ANALYSIS METHOD AND SELECTION OF VARIABLES

Analysis Tools and Procedures

The main tool used for this project was the track structural analysis computer program, KENTRACK, which was chosen as a model for the equations as noted in Chapter 2. This program is similar to the GEOTRACK program,[4] which has been used by the Association of American Railroads in its track research. The two programs model the track structure in a nearly identical fashion; the primary differences are in running time and output form.

Several hundred KENTRACK runs were made to determine the effect that different track structure variables had on the analysis parameters. The results of these runs were then used to select the variables to be included in each of the analysis equations.

An important step in this process was choosing the minimum and maximum values for each variable included in the analysis. The magnitude of the effect that each variable has on the analysis parameters depends greatly on the range of values assigned to that variable. Thus, a variable's apparent effect may be increased by increasing the range of values it takes. This apparent increased effect could potentially distort the practical influence of that variable. Therefore, in establishing the series of program runs, it was important to assign a range of values to each variable that would approximate the range expected to be encountered on Army track.

For each series of KENTRACK runs, only one variable was allowed to change in value--all other quantities and program related variables were held fixed. Each series consisted of a set of 5 to 14 runs in which the value of a single variable was gradually varied over its selected range, often, but not always, in uniform increments.

When the KENTRACK runs were completed, the output was subjected to two kinds of graphical analyses. The first type was primarily intended to illustrate the character of the behavior, or change, that each variable produced in the analysis parameters. These were simple, single-line graphs in which one variable was plotted on the horizontal (X) axis and the analysis parameter was shown on the vertical (Y) axis. Examples are shown in Figures 2 through 4.

One purpose of this graphical analysis was to observe where any portion of a variable range had an exceptionally great effect on the analytical parameter, or if any unexpected or unusual behavior occurred. This information was especially useful later when creating the test run data files, where a small number of values was required to represent the behavior over the entire variable range. When selecting these representative values, it was essential to note items such as the "steepness" of the curves over the variable range and points where changes in curve shape occurred.

A second type of graphical representation was used to illustrate the relative influence of each variable on the analysis parameters (Figures 5 through 9). In these figures, each vertical line represents one variable. The numbers at the ends of the lines are the maximum and minimum values used for that variable. Where room permits, intermediate values are also shown. The center horizontal line represents the reference case. This is the baseline from which values were varied (see next section). In the figures, the length of the vertical lines represents the relative influence that variable has on the parameter shown on the left-hand scale.

[4]C.S. Chang, C.W. Adegoke, and E.T. Selig, "The GEOTRACK Model for Railroad Track Performance," *Journal of Geotechnical Division, ASCE*, Vol 106, GT 11 (November 1980).

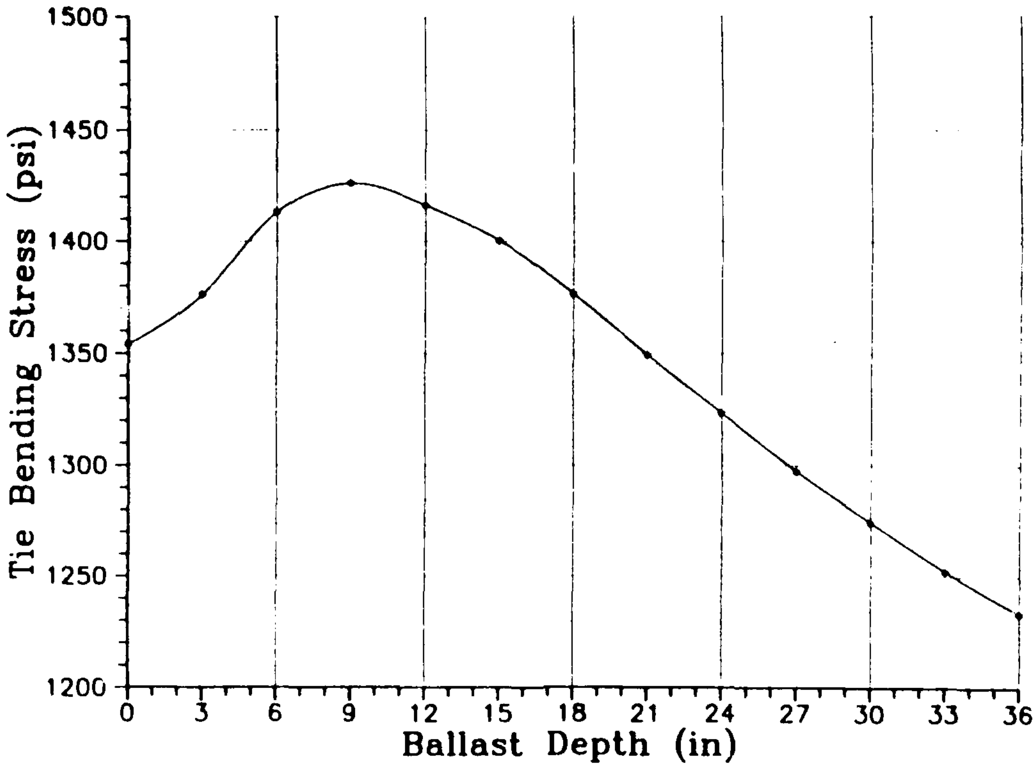

Figure 2. Ballast depth vs. tie bending stress (run 15).

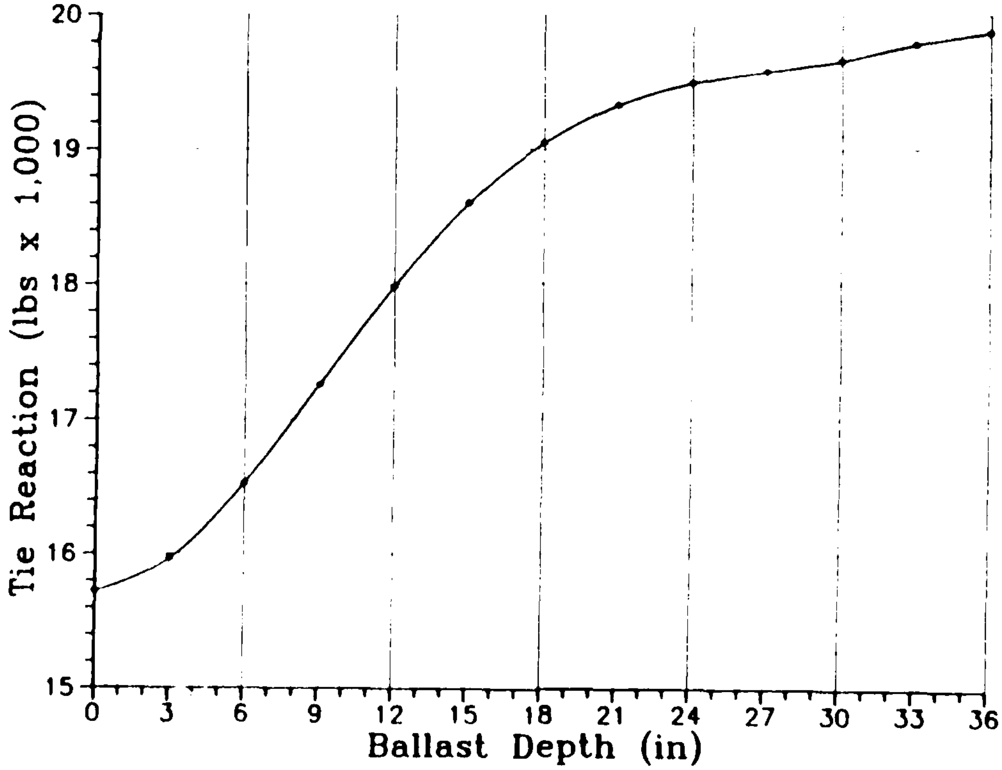

Figure 3. Tie reaction vs. ballast depth (run 15).

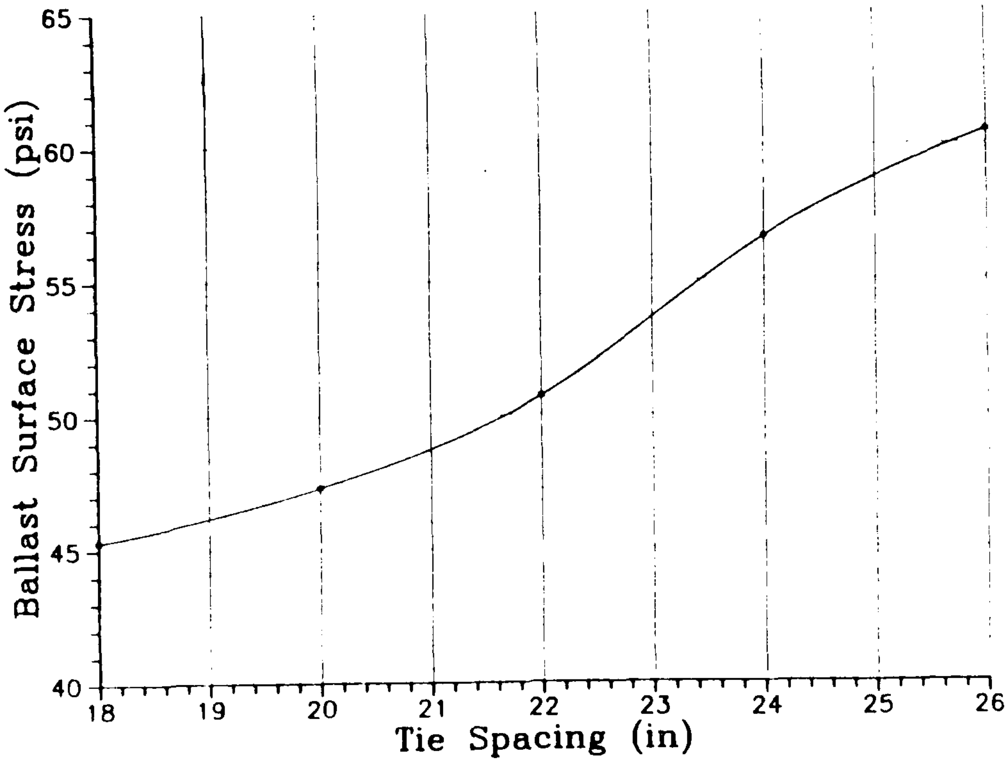

Figure 4. Tie spacing vs. ballast top stress (run 20).

As an example, refer to the Ballast Depth line in Figure 8. For the reference case (ballast depth = 12 in.) ballast surface stress is about 51 psi. If ballast depth changes to 6 in. (with all other values still set to the reference case), the surface stress is about 40 psi. In using these figures, it is important to note that only one variable can change at a time; all other values remain fixed at the reference case.

The Reference Case

Due to the complex interaction among the variables, it was useful to establish a reference case to serve as a base for comparison during development. The desirability of having this reference case can be seen when it is understood that the behavior any variable (e.g., ballast depth) produces in an analysis parameter (e.g., subgrade surface stress) depends on the values of all other variables.

While the choice of variable values for the reference case is not critical, it does have some influence on seeing the relative effects of each variable. Thus, it is helpful to have the reference case represent average situations or those that allow for easy visualization and comparison.

The reference track system represents common track construction at an Army installation, but with new rail and ties. The reference loading is that of a fully loaded six-axle 140-ton flat car traveling at 25 mph, with extra dynamic allowance for track and wheel irregularities. Except for rare situations, this is about the heaviest wheel load expected on Army track. Thus, this load serves as an effective reference since it is the adequacy of track under heavier loading that is of primary concern. Table 1 lists reference case values.

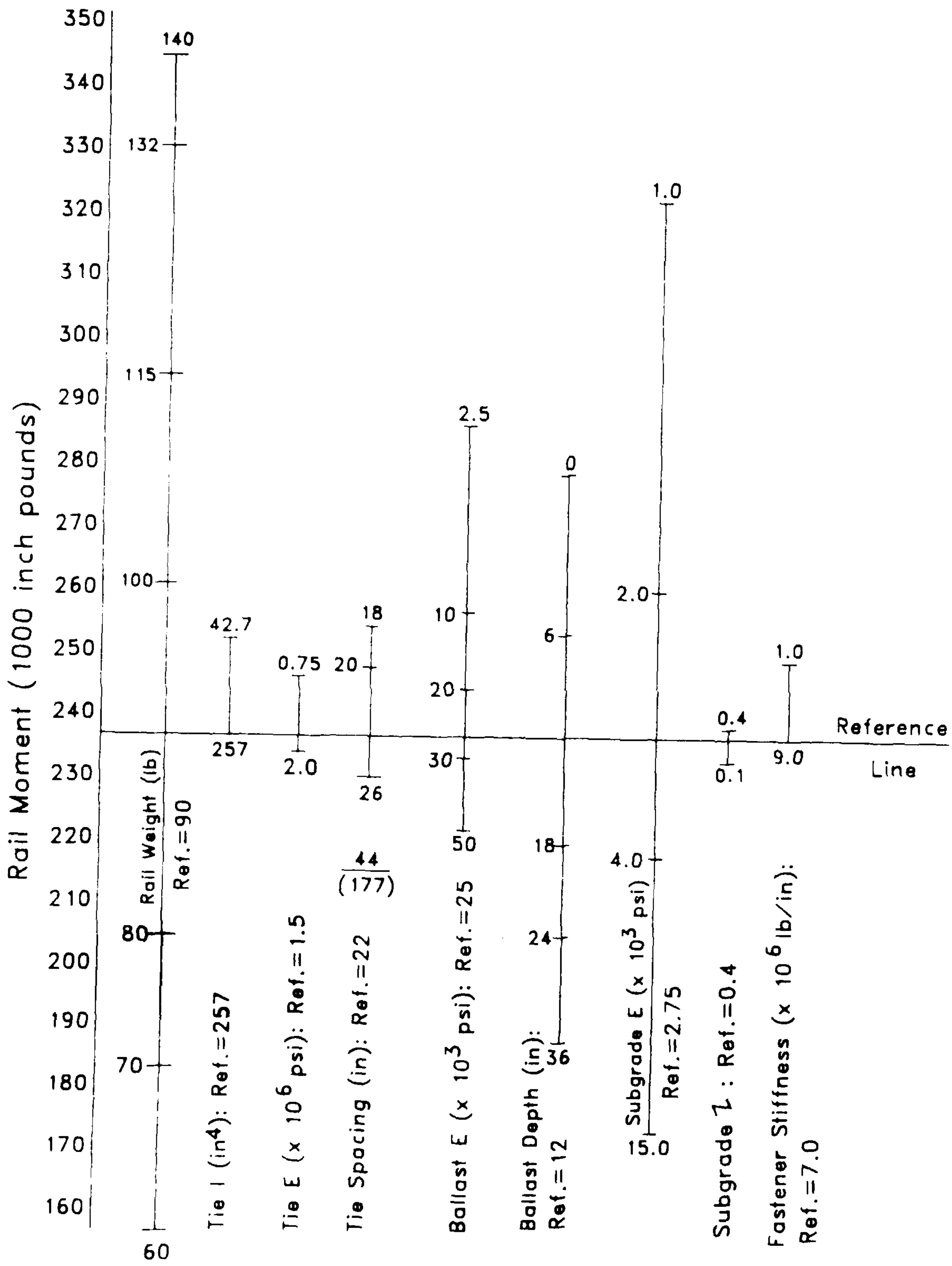

Figure 5. Effect of inputs on rail moment.

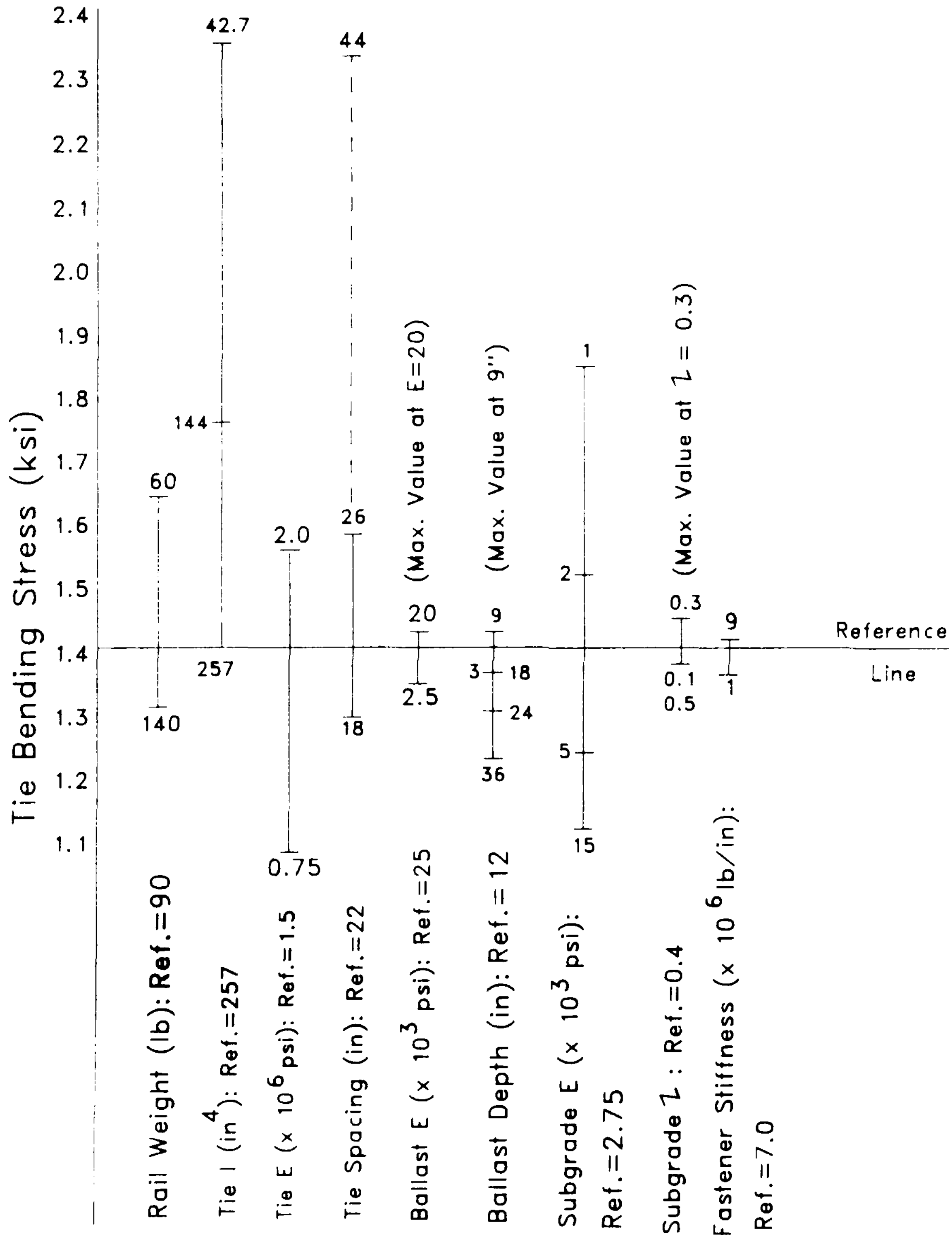

Figure 6. Effect of inputs on tie bending stress.

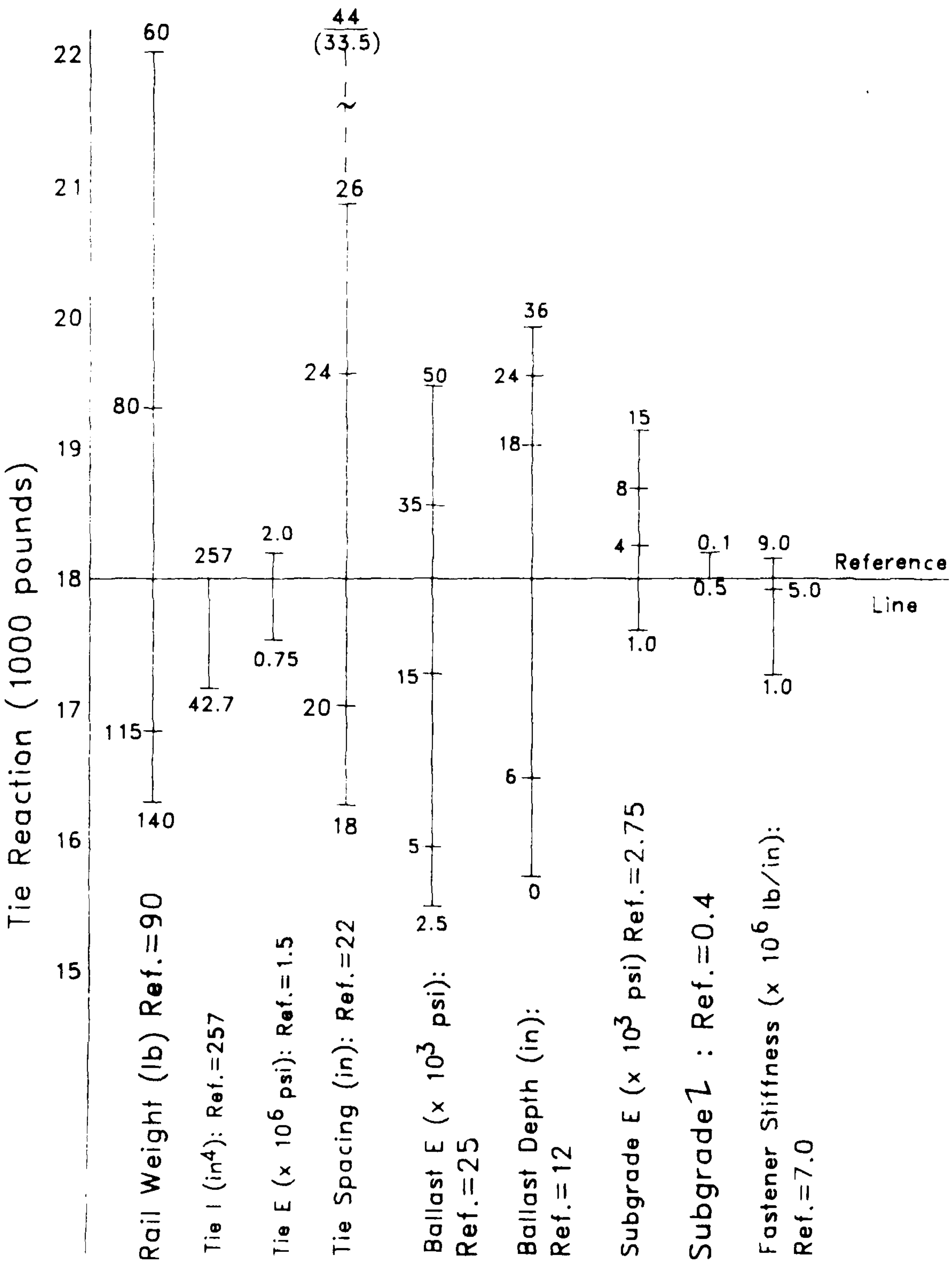

Figure 7. Effect of inputs on tie reaction.

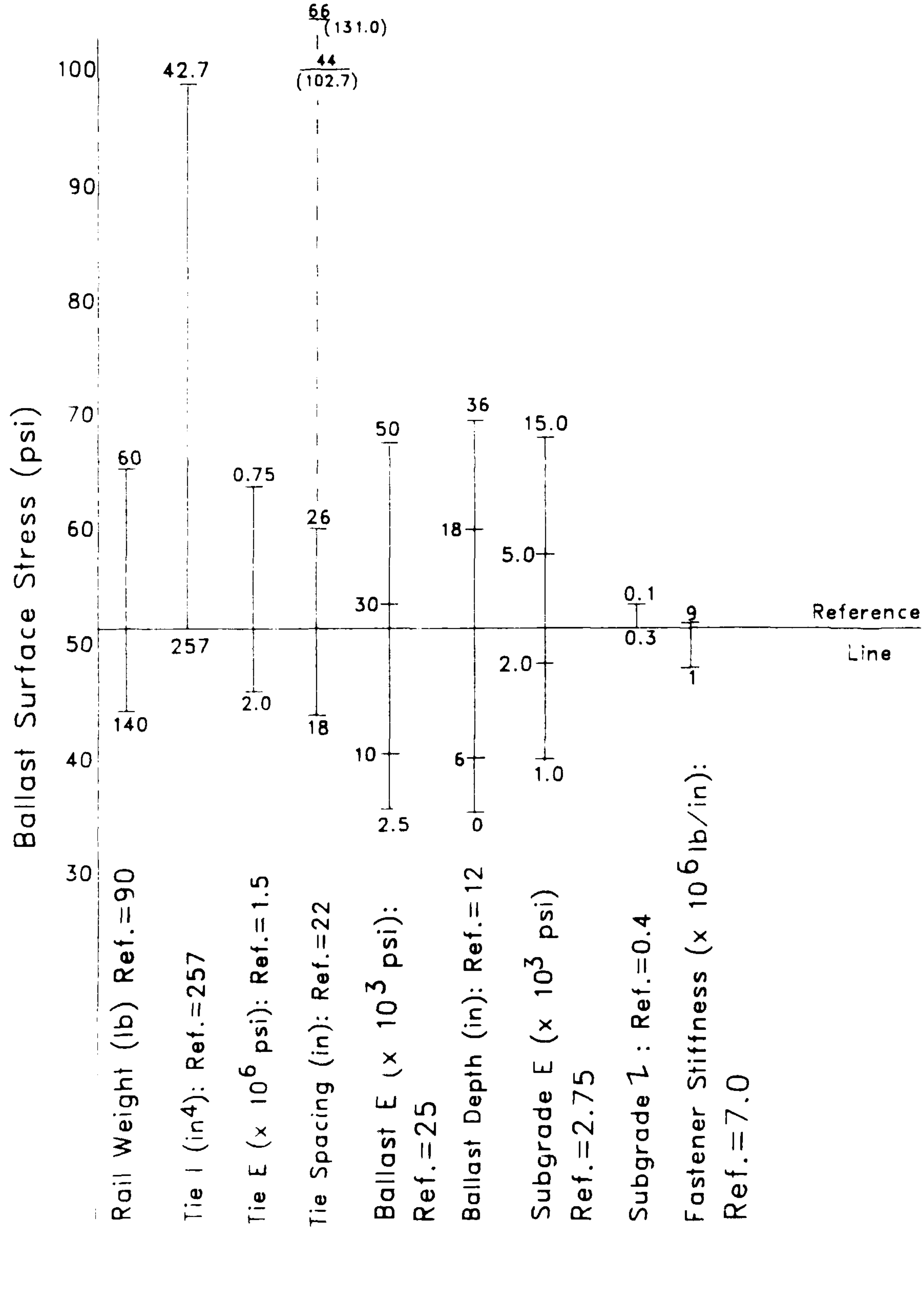

Figure 8. Effect of inputs on ballast surface stress.

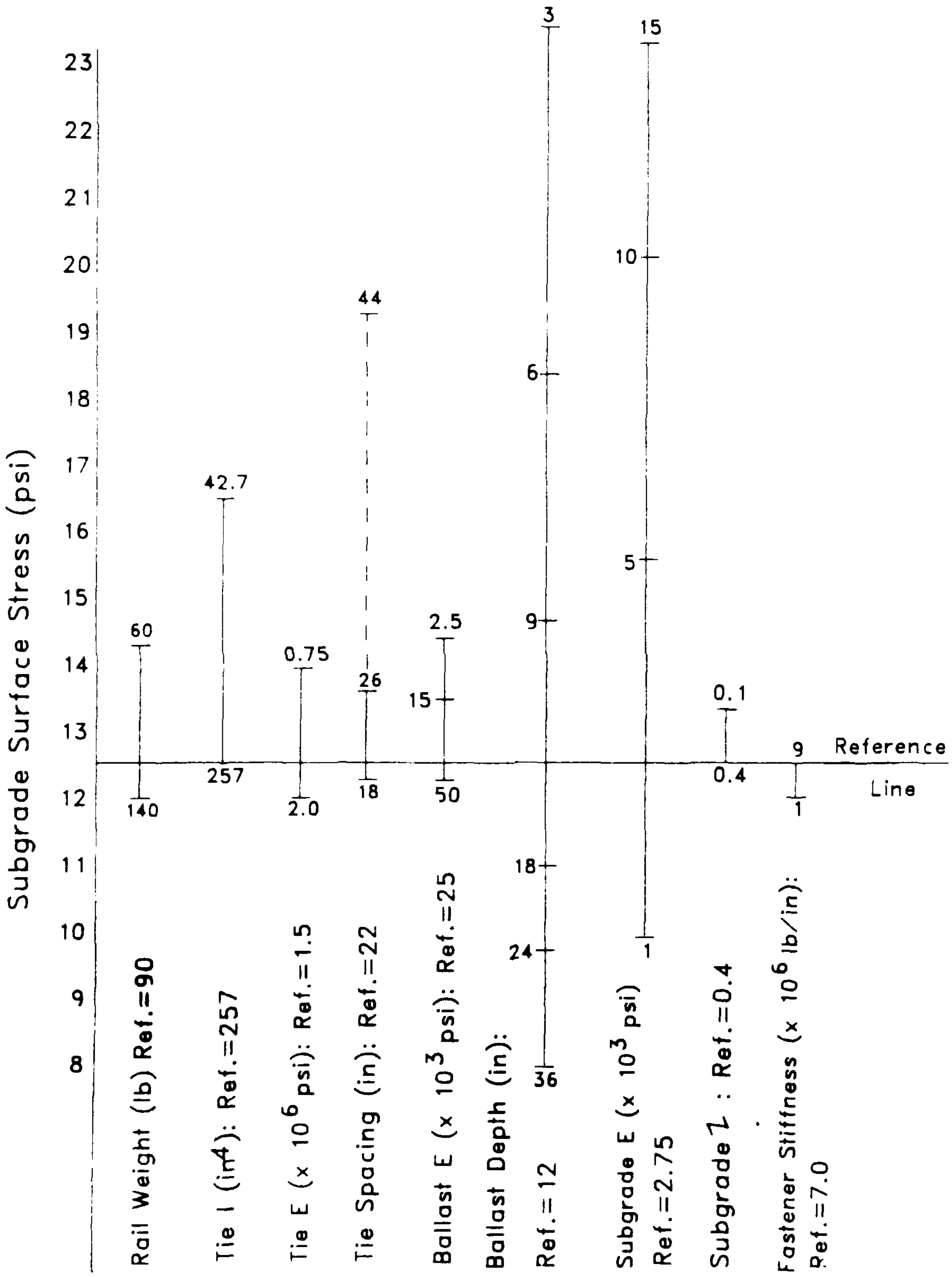

Figure 9. Effect of inputs on subgrade surface stress.

Table 1

KENTRACK Parameter Study--Reference Case Values

1. RSM (Rail Section Modulus - Base) = 15.2 $in.^3$
2. RMOI (Rail Moment of Inertia) = 38.7 $in.^4$ (90 lb - New)
3. RYM (Steel E) = 30 x 10^6 psi
4. RTK (Rail/Tie Spring Constant) = 7 x 10^6 lbs/in.
5. THIGH (Tie Thickness) = 7 in.
6. TMOI (Tie Moment of Inertia) = 257.25 $in.^4$
7. TWID (Tie Width) = 9 in.
8. TSPA (Tie Space) - 22 in.
9. TYM (Tie E) = 1.50 x 10^6 (Oak-Hazard, See AREA p. 7-2-28, 1988)
10. NONUT (Nonuniform Tie) = 0 (No)
11. NPTD (Transverse Nodal Points) = 7
12. LNRT (Rail Nodal Point No.) = 4
13. Y(NPTD) (Nodal Point Locations) = 0, 15, 25.75, 36.5, 47.25, 58, 66
14. NLOAD (Number of Wheel Loads) = 3
15. NMOUT (No. Months for Output) = 1 (or 0 for Summary)
16. NTA (No. Ties for Analysis) = 6
17. NMA (No. Months for Analysis) = 1
18. IDAMA (Damage Analysis) = 0 (No)
19. NLBT (No. Layers for Tensile Strain) = 0
20. NLTC (No. Layers for Compressive Stress) = 1-4
21. NBT (Beginning Tie for Analysis) = 2
22. NET (End Tie for Analysis) = 11
23. NXOUT (No. Cross Sec. for Output) = 1-9
24. QxD(I) (Load Locations) = 44 in., 110 in., 176 in.
25. Q(I) (Magnitude of Loads) = 40,000 lbs ea.
26. MNOUT(NMOUT) (No. Months Output) = 1
27. IXOUT(NXOUT) (Cross Sections for Output) = (as needed) (can vary)
28. NLS (No. Layers) = 3-5 (5 is max/run)
29. NLTEMP (No. Asphalt Layers) = 0
30. TORVD (Tolerance for Vertical Moment) = 0.0001
31. TORTS (Tolerance for Tensile Strain) = 0.0100
32. K1(I) (Modulus of Elasticity for each layer): See chart below
33. K2(I) (Nonlinear exponent for K1(I) = 0 (All layers are assumed to behave linearly)
34. PR(I) (Poisson's ratio for each layer): See chart below
35. HIGH(I) (Thickness of Layers): See chart below
36. LNTC(NLTC) (No. of Layers for Compressive Stress at Top of layer) = variable, 2-5 (Values for layer 1 are not valid)

	No. 32	No. 34		
Layer No.	**K1(I)**	**PR(I)**	**HIGH(I)**	**Layer Type**
1*	25,000	0.35	12 in.	Average Ballast (Partly Fouled)
2*	2,750	0.40	228 in.	Med. Soft Subgrade
3**	10^{20}	0.50	----	Rigid Layer

Wheel Load Configurations

The fully loaded 140-ton flat car was used as a reference load in developing the equations. It served this purpose well since there has been much concern about handling this car on Army installations, and it represents the heaviest load commonly found on Army track.

As a reference load, however, this car does have one drawback: it has three-axle trucks whereas most standard freight cars have two-axle trucks. Figure 10 is a diagram of two-axle and three-axle trucks. The concern was that the maximum track stresses produced by the three-axle truck would be different than those produced by the standard two-axle truck due to the presence of an additional axle.

Several series of KENTRACK runs were made and the results analyzed to determine the possible additive effects of the third axle. These runs were done in an attempt to maximize any adjacent wheel effects.

The analysis showed that the extra axle did produce some additive effects, but these were usually small enough that they could be ignored for the intended use of the equations. The results suggested that, for two-axle trucks with dynamic wheel loads greater than 35,000 lb on track with ballast less than 6 in. and where subgrade is medium soft or softer, the equation results should be multiplied by 0.9 to obtain the best match with results from the KENTRACK program. However, it appeared reasonable to conclude that in most cases, the results from the equations would be about equally accurate for cars and engines with either two- or three-axle trucks.

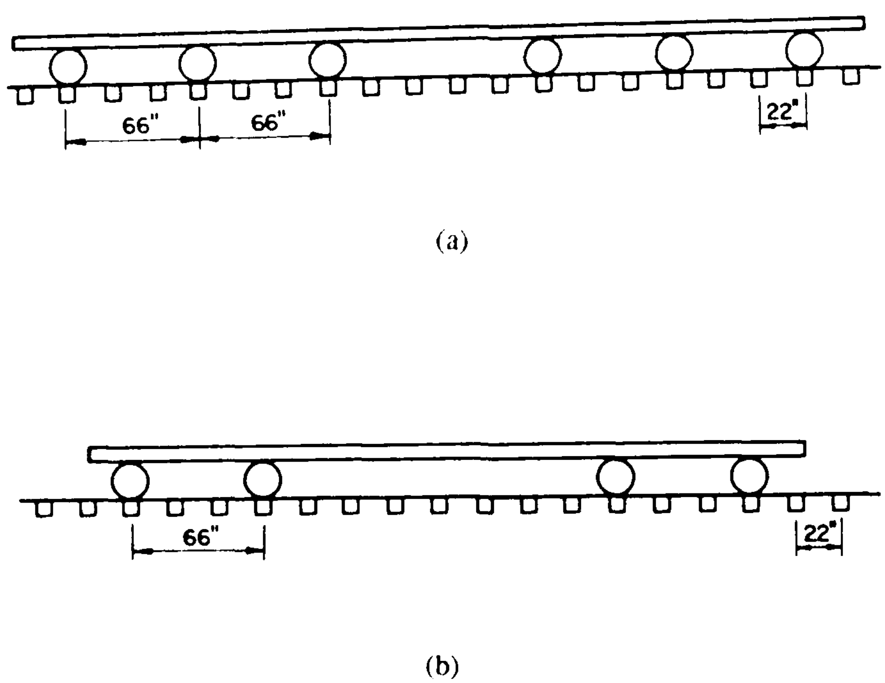

Figure 10. Rail car with (a) three-axle trucks and (b) two-axle trucks.

Track-Related Variables

There were two major conflicting objectives involved in selecting the variables to be included in the equations. First was the desire to maximize the accuracy with which the equations matched the KENTRACK results. Obtaining accuracy meant including the largest number of variables. Second was the need to reduce complexity enough to work successfully with the data and produce the equations. This requirement called for using the fewest number of variables.

Resolving this conflict required careful study of the effects each variable had on the analysis parameters and using judgment in deciding how and where compromises should be made. Figures 5 through 9 were especially useful in this process.

In the equations for ballast surface stress and subgrade surface stress, it proved possible to combine two variables--tie modulus of elasticity (tie E) and tie moment of inertia (tie I)--into one variable: the two quantities were multiplied together (tie EI). A series of KENTRACK runs was made to study the effect of this combination and the results indicated that this combination could be done without unacceptable loss of accuracy. By combining the two variables into one, the difficulty in developing the two surface stress equations was reduced.

At this point in the project, it was not known if equations with acceptable accuracy could be produced from the selected group of variables, so the variable choice was considered tentative. If needed, variables would be added later to ensure high enough accuracy.

The Wheel Load Variable

Noticeably absent from Figures 5 through 9 and Tables 2 and 3 (page 30) is the critical wheel load variable. Initial KENTRACK runs made with wheel loads varying from 5,000 to 50,000 lb indicated that the track component stresses and loads varied linearly with wheel load. Thus, the wheel load variable could be handled simply as a multiplier of the equation results.

As a convenience during development, the wheel load was kept at the reference value of 40,000 lb. The effect of different wheel loads is then obtained by using a multiplier of the desired wheel load divided by 40,000.

As with the other reference values, the choice of the reference wheel load (at 40,000 lb) had no great effect on the results obtained with the final product--only on its form of presentation. The equations could have been developed just as well with the wheel load set at 1, with the equation results simply mulitiplied by the wheel load.

The Five Analysis Parameters and Their Variables

After study and analysis of several hundred KENTRACK runs, five analytical parameters were selected, with the number of variables to be included in their equations ranging from 4 to 7. Table 2 lists these parameters and their related variables.

Note that rail moment is the parameter for the rail analysis shown in Figure 5. This parameter was later converted to maximum rail vertical bending stress, which appears in the rail equation.

For ties, there were two important indicators: tie bending stress and tie reaction. Tie bending stress indicates the maximum vertical bending stress along the tie's length, whereas tie reaction indicates the

Table 2

Analytical Parameters for Equation Development

RAIL VERTICAL BENDING STRESS

Rail Weight - 60, 75, 90, 115, 132 lb/yd
Ballast Depth - 3, 12, 21, 30 in.
Subgrade Modulus of Elasticity - 1500, 2750, 5000, 10000 lb/sq in.
Ballast Modulus of Elasticity - 5000, 15000, 25000, 40000 lb/sq in.

TIE BENDING STRESS

Tie Spacing - 22, 44, 66 in.
Tie Modulus of Elasticity - 0.75×10^6, 1.25×10^6, 2.0×10^6 lb/sq in.
Subgrade Modulus of Elasticity - 1500, 2750, 10000 lb/sq in.
Rail Weight - 60, 75, 90, 132 lb/yd
Tie Moment of Inertia - 42.70, 94.00, 144.00, 257.00 $in.^4$

TIE REACTION

Tie Spacing - 22, 44, 66 in.
Rail Weight - 75, 90, 115, 132 lb/yd
Ballast Depth - 3, 12, 21, 30 in.
Ballast Modulus of Elasticity - 10000, 25000, 40000 lb/sq in.
Subgrade Modulus of Elasticity - 1500, 5000, 10000 lb/sq in.

BALLAST SURFACE STRESS

Tie Spacing - 22, 44, 66 in.
Tie Modulus of Elasticity x Moment of Inertia - 32.0×10^6, 94.0×10^6, 216.0×10^6, 386×10^6 $lb\text{-}in.^2$
Ballast Depth - 3, 12, 21, 30 in.
Ballast Modulus of Elasticity - 10000, 25000, 40000 lb/sq in.
Subgrade Modulus of Elasticity - 1500, 2750, 5000, 10000 lb/sq in.
Rail Weight - 75, 90, 115, 132 lb/yd

SUBGRADE SURFACE STRESS

Ballast Depth - 3, 12, 21, 30 in.
Subgrade Modulus of Elasticity - 1500, 2750, 5000, 10000 lb/sq in.
Tie Spacing - 22, 44, 66 in.
Tie Elastic Modulus x Tie Moment Inertia - 32×10^6, 94×10^6, 216×10^6, 286×10^6, 386×10^6 $lb\text{-}in.^2$

vertical load (force) on the tie, and therefore, the amount of wheel load supported by that tie (as opposed to adjacent ties). The two tie equations cannot be combined because their variables do not match. In addition, their common variables affect them not only in different degrees, but sometimes in the opposite manner. In Figures 11 and 12, for example, as the subgrade modulus increases, tie reaction also increases but tie bending stress decreases.

Ballast and subgrade each have an equation representing the maximum stress on their top surface. (For ballast, the top surface is taken at the bottom face of the tie.)

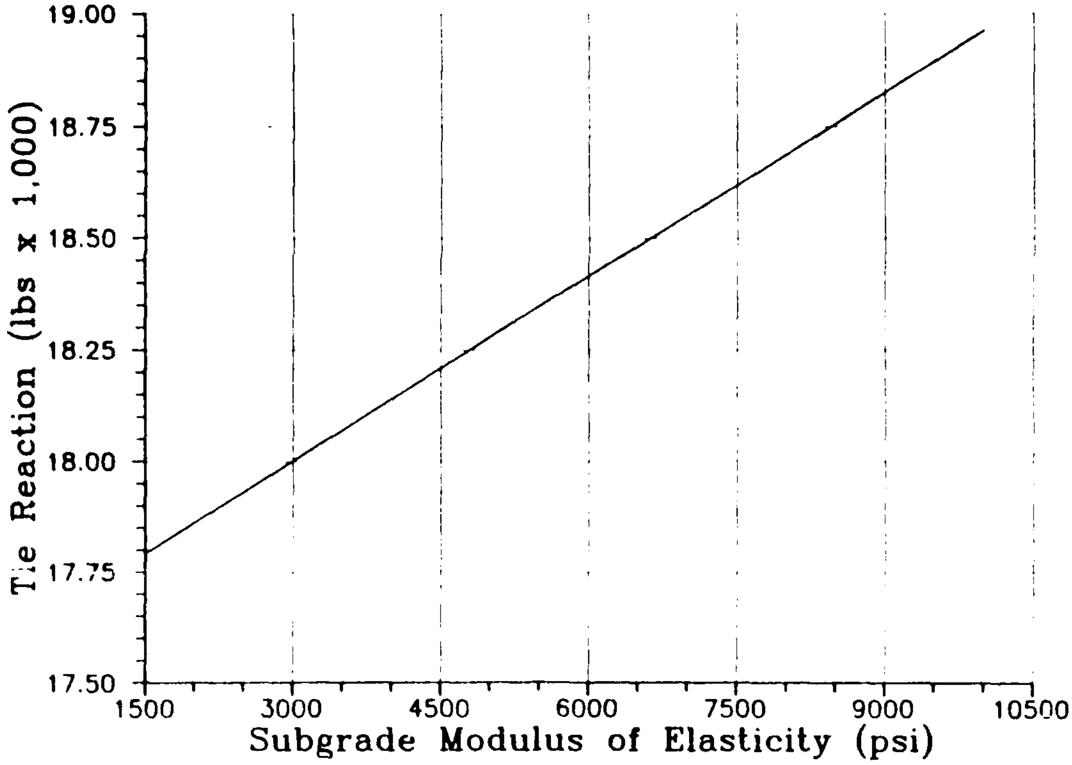

Figure 11. Tie reaction vs. subgrade modulus.

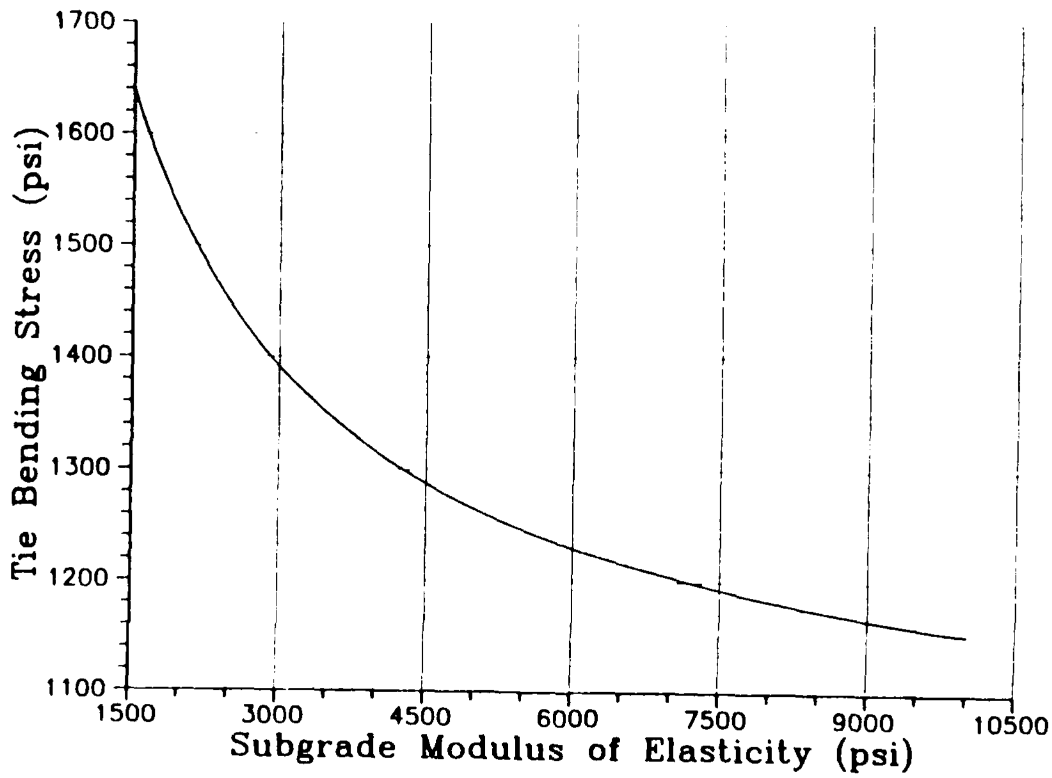

Figure 12. Tie bending stress vs. subgrade modulus.

4 DEVELOPMENT OF THE EQUATIONS

Obstacles to Development

An accurate mathematical model of track structure behavior under load requires the use of methods such as Burmister's layered theory and finite element analysis, two methods used by the KENTRACK program[5]. Computations for these methods require an extensive computer program to handle the required stiffness matrices. Thus, there are no simple expressions for stresses in the track components that can be extracted from the program and used for basic structural analysis.

An additional difficulty encountered in modeling the behavior of the track structure under load is in accounting for the interaction of track components. This interaction means that, in any simple mathematical expression for the stress in the track components, the main variables will not be independent. That is, the effect that any variable has on the stress in a component will depend on the values of the other variables.

This complex behavior is illustrated in Figures 13 through 15. Figure 13 is a simple graph of the effect that ballast depth has on the value of ballast surface stress (with all other variable values set at the reference case). In Figure 14, this same relationship is shown, with the additional effects of changing the ballast modulus. When the ballast modulus is 25,000, changes in ballast stress with changes in ballast depth are illustrated by line 2--the reference case. However, when the ballast modulus is either 10,000 or 40,000, the relationship between ballast stress and ballast depth is given by line 1 or 3, respectively. In general, the three lines appear to diverge from the point representing a ballast depth of 3 in. and a ballast stress of 35 psi. Clearly, it is not possible to determine how changes in ballast depth will affect ballast stress without also knowing the value for ballast modulus.

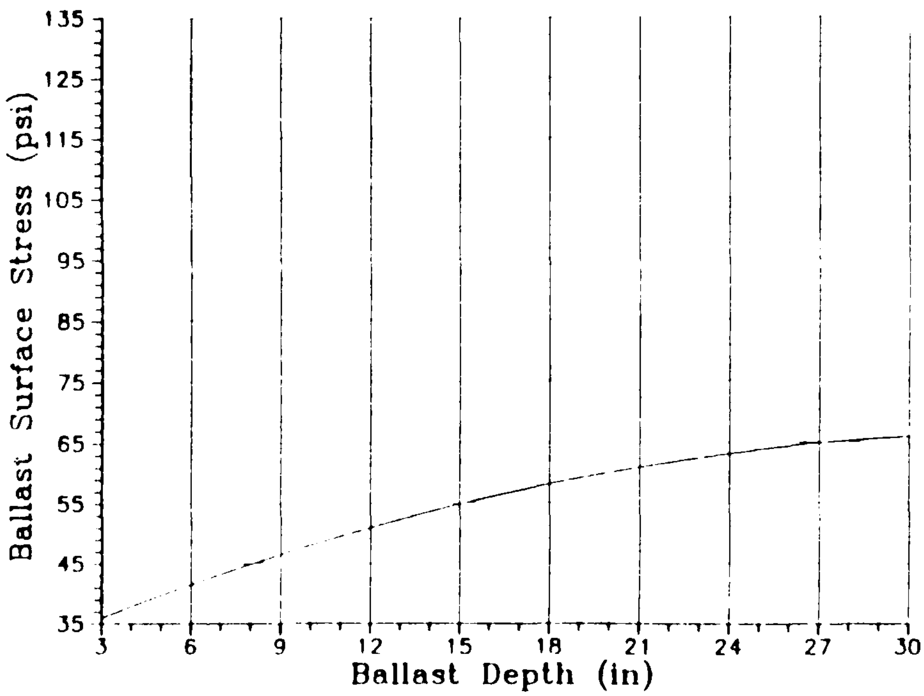

Figure 13. Ballast stress vs. ballast depth.

[5] Y.H. Huang, et al.

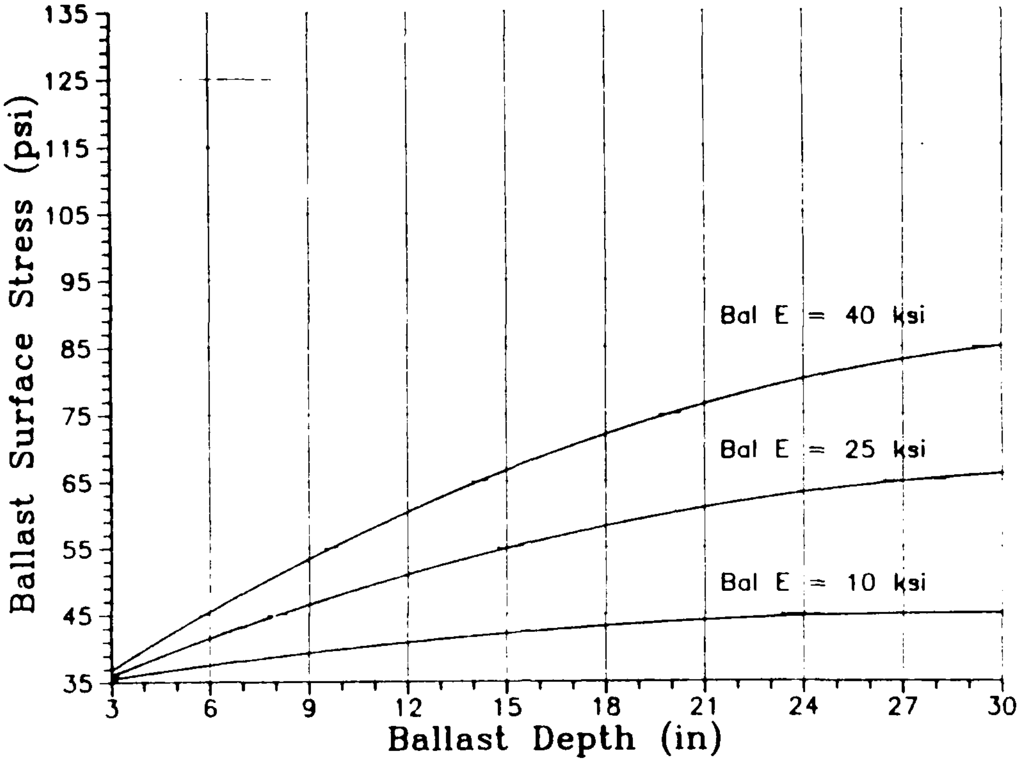

Figure 14. Ballast stress vs. ballast depth when ballast modulus is varied.

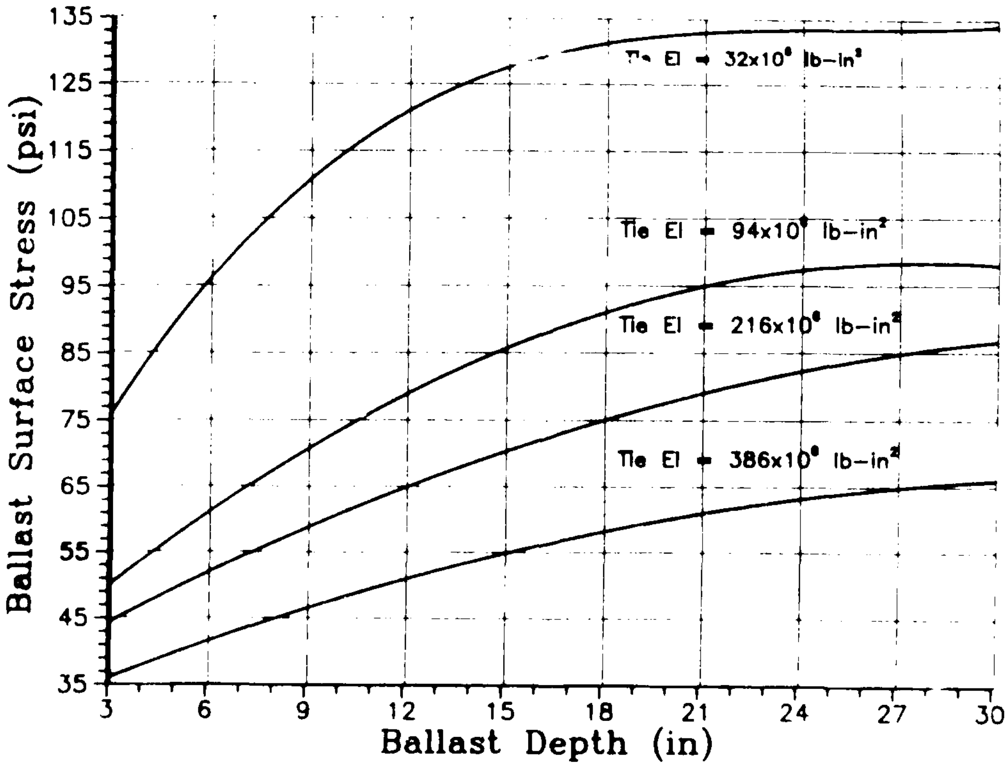

Figure 15. Ballast stress vs. ballast depth when tie EI is varied.

Taking this illustration one step further, refer to Figure 15. In this graph, the reference line is at the bottom, identical to the line shown in Figure 13 and also to the middle line in Figure 14. In this case, the values for tie EI (tie modulus x tie moment of inertia) are varied. As tie EI decreases, the lines shift upward and increase in curvature. Thus, the value for tie EI must also be known before the relationship between ballast surface stress and ballast depth can be determined.

This same phenomenon occurs with the remaining variables in the ballast stress equation; their values must also be known before the relationship between ballast surface stress and ballast depth can be defined. Also, as with ballast depth, a similar interrelationship occurs with the other variables in the ballast stress equation, and in fact, for all variables in all of the equations. Thus, as an analysis tool, the conventional approach of varying one variable while holding all others fixed does not work well.

To increase efficiency, the research turned toward finding methods that would help in developing the structural analysis equations. During this search, USACERL contacted the Departments of Statistics and Mechanical and Industrial Engineering at the University of Illinois for assistance. Their response confirmed that the complexity of the relationships among the variables precluded the use of any known conventional analytical methods in determining single, simple equations to indicate the stress in the track components. Existing analytical methods could serve only as rough indicators as to what form the equations should take.

Development of the equations required a concentrated effort in which several analytical methods were used in combination, along with engineering judgment about the behavior of the stresses as the variables changed in value. The rest of this chapter describes the basic procedures used in developing the equations.

Creating the Data Files

Since no expressions could be extracted from the KENTRACK program to serve as simplified analysis equations, the next best choice was to create equations, based on the behavior of the KENTRACK program. A representation of this behavior was created by producing a data set for each of the five analytical parameters.

The data sets were established by obtaining the KENTRACK-produced values for the full range of track conditions expected to be encountered. Values for each of the equation variables were changed, in steps, one at a time, until the whole range was covered. This data set then became the data file used as a basis for creating the equations.

Because each change in variable value resulted in another run of the KENTRACK program, it was clear that choosing a large number of values (or steps) for each of the variables would result in several thousand output values for each of the data sets. This process would result in an unwieldy data file. Thus, another phase of analysis was required to select a representative set of values for each variable.

Choosing the number of values for each variable was clearly a tradeoff. A large number of values was desirable because it would ensure good conformity with the KENTRACK results, but a smaller number was necessary to keep the data file to a manageable size. So, the challenge was to select the fewest values that would permit adequate representation of each variable's behavior.

Another consideration in choosing the values was to capture the character of the variable behavior, not just the range. First, the extreme values were established for the normal range of a variable. Then the middle values were chosen such that they best captured the relationship. Figure 16 shows an example of how this choice was made. The four marked points were used to define the shape of the curve.

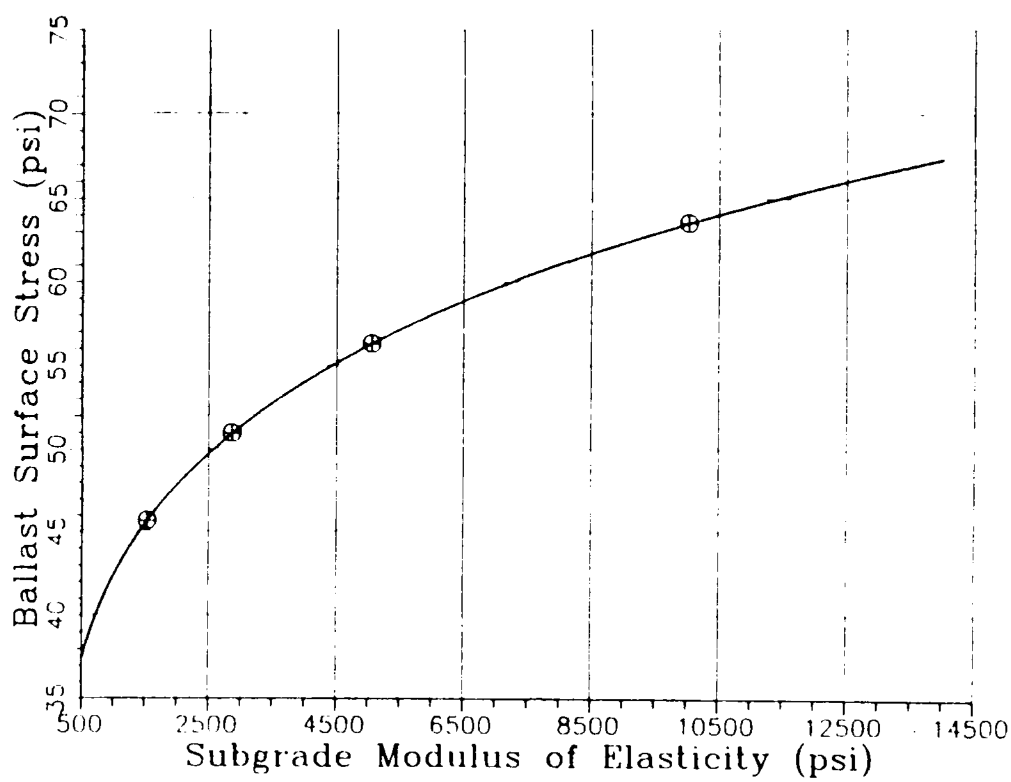

Figure 16. Ballast surface stress vs. subgrade modulus.

Table 2 summarizes variables and values chosen for each parameter. Once the values were chosen, KENTRACK runs were made with all possible combinations of variable values.

Table 1 lists the values chosen for other input to the KENTRACK program. Most of these values represent requirements for operating the program. The few that are track system variables were determined from previous analysis to have little effect on the five analytical parameters compared with the selected equation variables.

While one KENTRACK run was needed for each combination of variable values for each of the five equations, there was an additional requirement for the subgrade stress, tie reaction, tie bending stress, and ballast surface stress data files. This occurred when the variable "tie spacing" was set to 44 in. In this situation, two sets of KENTRACK runs were needed to ensure that the parameters' maximum output values were being used in the data files.

The reason for the extra set of runs relates to the way the KENTRACK program functions. KENTRACK gives output only for points along the centerlines of a tie. Since the wheel loading used to produce the data files was that of a car with three-axle trucks and a 66-in. wheel spacing, the middle and outer wheels could not both be positioned over a tie when tie spacing was 44 in. (Figure 17). So, two runs of KENTRACK were made, one with the center wheel positioned over a tie and a second with an outer wheel positioned over a tie. Then the largest output value of the two runs was used in the data file.

Figure 18 is a sample data file for subgrade stress in matrix form. Table 3 summarizes the number of KENTRACK runs made for each parameter and the number of values chosen for each variable.

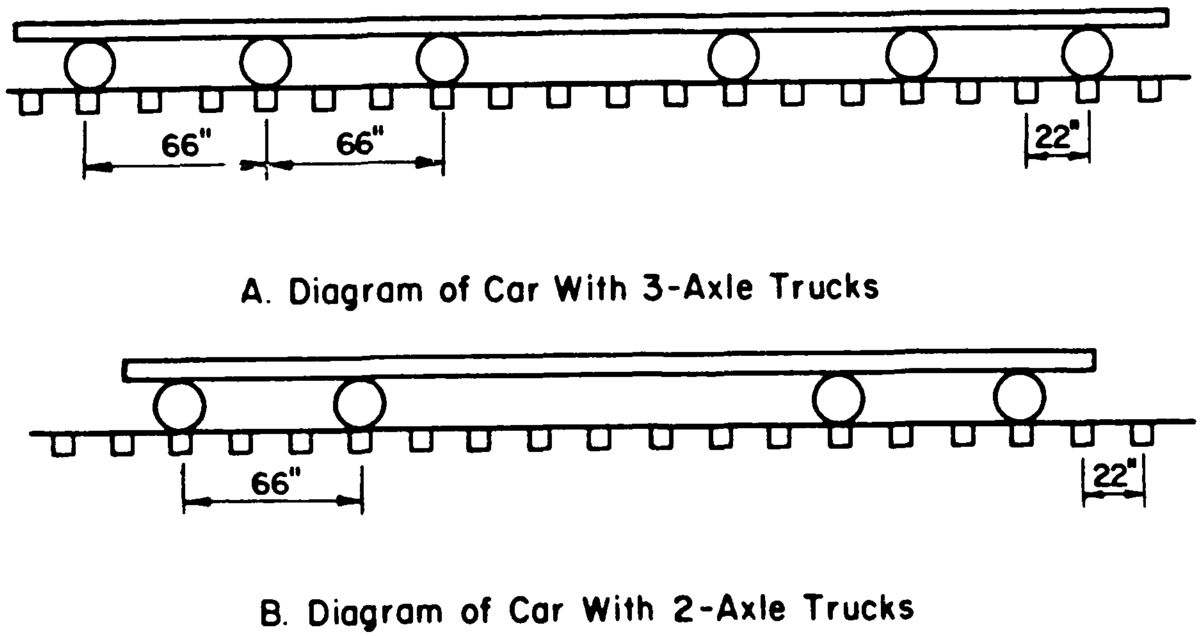

Figure 17. Wheel loading for car with three-axle trucks: (a) output under tie nos. 2 and 5 and (b) output under tie no. 4.

Analytical Methods

Due to the complex behavior of the parameters, a combination of several analytical methods was required in developing the equations. Most methods employed computer software: Energraphics for graphical methods,[6] FACT for factorial analysis,[7] Statistical Package for the Social Sciences (SPSS) for linear regression and statistical analysis,[8] and CHECK, a program written in-house to compare the output from each equation attempt with the KENTRACK data set. These methods were used in an iterative process that varied somewhat for each equation. The general process is described below.

Once the primary variables and representative variable values were selected, and KENTRACK output data files were established, families of graphs were plotted for each variable and parameter combination. Each graph had multiple lines, with one equation variable on the X-axis and the analytical parameter on the Y-axis. The multiple lines were produced by first plotting the behavior of the primary (X-axis) variable with all other values set at the reference case. Then, the value of a selected second variable was changed and the primary variable behavior replotted. This process was repeated for each selected value of the second variable. The same graphs were then replotted with X, Y, and then both axes on a logarithmic scale. Most of the graphs contained three or four lines. Figures 19 through 22 depict a family of graphs.

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[6] *Energraphics 2.0 Beginner's Guide* (Enertronics Research, Inc., 1985).

[7] J. Kim, *Factor Analysis* (Sage Publications, 1978).

[8] N.H. Nie et al., *Statistical Package for the Social Sciences*, 2nd ed. (McGraw-Hill, 1975).

Ballast Depth		3					12					21					30				
Tie EI x 10E6		32	94	216	286	386	32	94	216	286	386	32	94	216	286	386	32	94	216	286	386
Tie Spacing	Subgrade Modulus																				
22	10000	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
		802	616	495	461	429	281	248	219	210	200	154	144	133	130	126	114	110	105	104	103
	5000	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
		602	442	351	328	307	221	193	174	162	155	133	125	117	115	112	101	98	96	96	95
	2750	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
		457	331	268	253	239	178	155	137	131	125	116	111	105	103	102	90	89	88	87	87
	1500	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
		335	246	205	196	187	148	132	118	114	109	102	98	95	94	93	79	78	77	77	76
44	10000	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
		1735	1319	1038	959	885	509	437	376	355	335	233	215	197	190	183	146	141	134	132	129
	5000	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220
		1311	975	774	722	676	375	324	280	266	252	183	171	159	155	150	122	118	114	113	113
	2750	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
		986	729	594	561	533	280	243	213	203	193	147	140	132	129	126	102	100	99	99	98
	1500	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260
		712	535	451	431	414	208	184	165	158	152	119	115	110	109	108	85	84	84	83	83
66	10000	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280
		2212	1655	1285	1182	1086	633	537	456	430	403	274	249	223	215	206	161	152	143	140	136
	5000	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300
		1737	1274	1000	931	869	473	404	346	328	310	210	193	176	171	165	130	124	119	116	116
	2750	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320
		1345	986	798	753	714	353	303	262	250	237	164	153	143	139	135	107	104	101	96	99
	1500	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340
		996	746	628	601	578	258	226	199	191	183	129	122	116	113	112	87	85	85	84	84

Top number: 101 : Run number
Bottom number: 802 : Subgrade stress x 10

Figure 18. Sample data file for subgrade stress equation.

Table 3

KENTRACK Runs for Each Parameter and Number of Values Chosen for Each Variable

Variable	Rail Bending Stress	Tie Bending Stress	Tie Reaction	Ballast Stress	Subgrade Stress
Rail Weight	5	4	4	4	
Subgrade Modulus	4	3	3	4	4
Ballast Depth	4		4	4	4
Ballast Modulus	4		3	3	
Tie Spacing		3	3	3	3
Tie Modulus		3			
Tie Moment of Inertia		4			
Tie Modulus Tie Moment				4	5
Minimum No. of KENTRACK Runs Required	320	432	432	2304	240
With Center and End Wheels over a Tie		144	144	768	80
Total	320	576	576	3072	320

The number of variables selected for the equation determined the number of graphs needed for each parameter. For instance, 80 graphs were required for tie bending stress. Five variables were chosen to model this parameter. With tie bending stress on the Y-axis, each of the five was plotted individually with one of the remaining four as a secondary variable (5 x 4 = 20 combinations). Each of these graphs was replotted with the X, then Y, and then both axes on a logarithmic scale (20 x 4 = 80 graphs).

The analysis methods available for this study worked best when the data could be expressed in linear form. In addition, a linear form generally made it easier to examine and grasp the complex interrelationships among the variables. Thus, the primary reason for producing the families of graphs was to determine if the behavior of the variables could be "forced" into approximate straight-line relationships--by choosing combinations of natural and logarithmic scales.

Again, compromises were required. While each variable within an equation could be expressed in a different form, the analytical parameter (dependent variable) could, of course, take only one form. Typically, some graphs showed a better straight-line relationship when the analytical parameter was in natural form, whereas others were better with the parameter in log form. The form chosen for the analytical parameter was the one that allowed the largest number of equation variables to exhibit the best approximation of straight-line behavior. This choice, however, was not always obvious.

Another factor was considered in choosing the form of the analytical parameter--the relative effect or "importance" of each variable within the equation. This relative effect was obtained from results of factorial and statistical analysis runs. Thus, in selecting the form of the analytical parameter, the objective was to select the one that resulted in the best straight-line relationships with those variables having the greatest effect on the parameter.

The next step in development involved factorial analysis. This analysis was used to determine the most important terms (variables) for modeling each parameter in its chosen form. The program output listed single variables and multiple variable combinations in order of their ability to affect the value of the analytical parameter. Thus, the results would be a first cut at selecting the number of terms used in each equation. Figure 23 shows this list for tie reaction. A similar list was produced for each parameter.

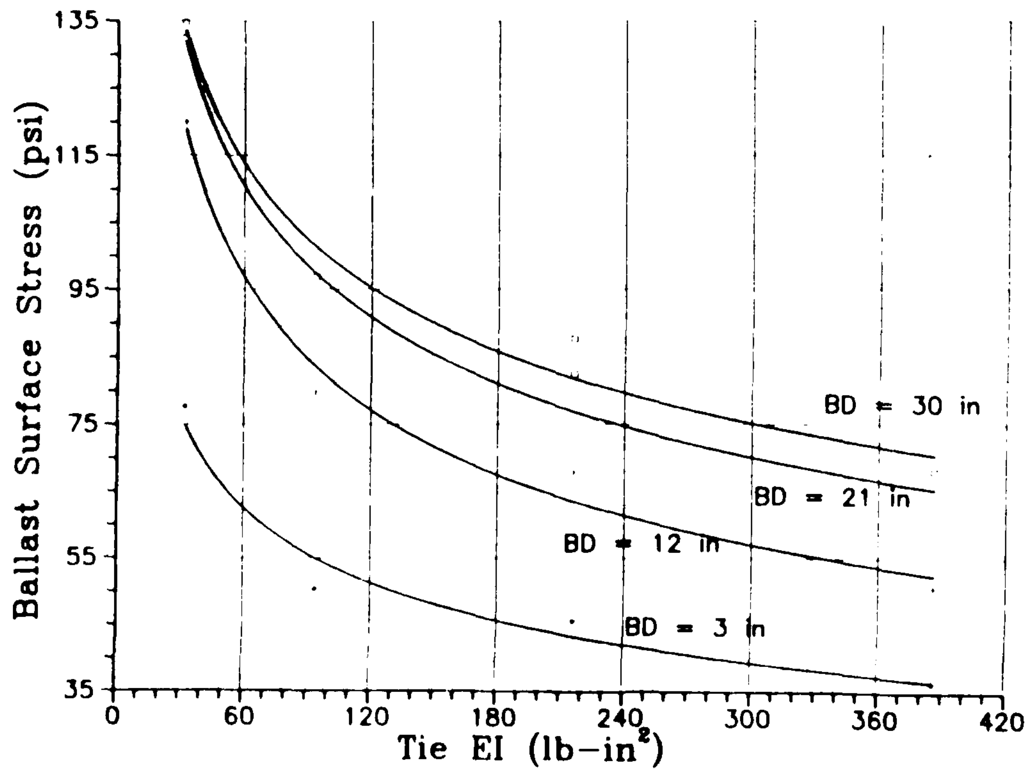

Figure 19. Ballast surface stress vs. tie EI when ballast depth is varied.

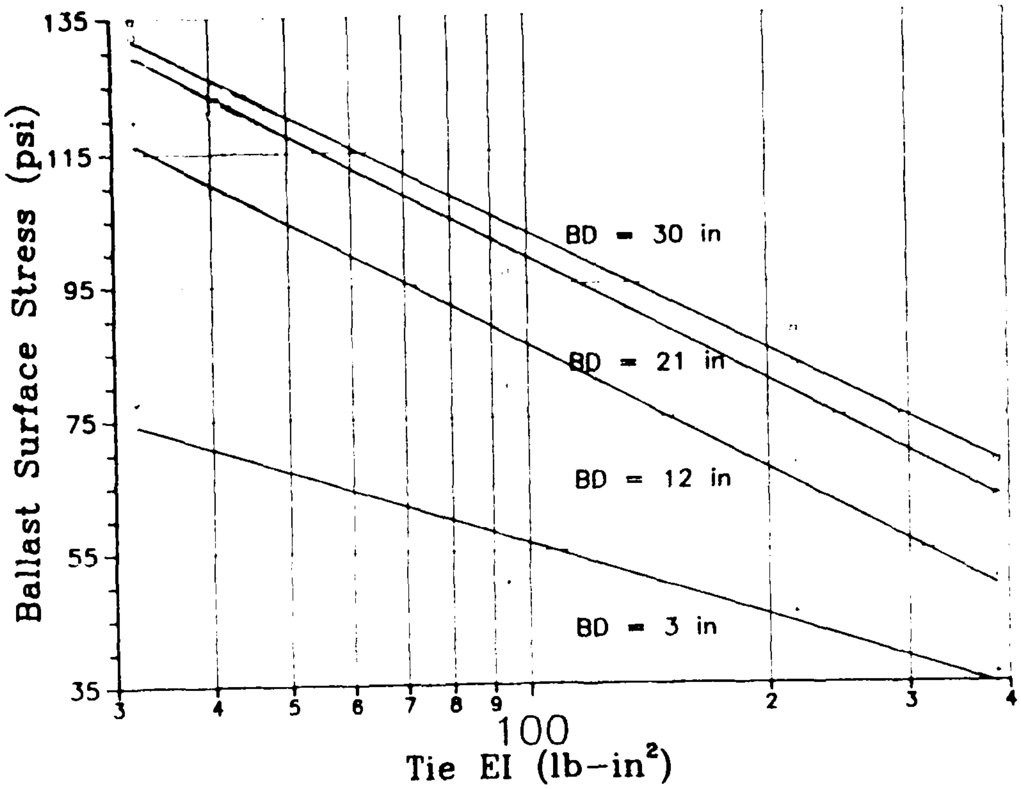

Figure 20. Ballast surface stress vs. log tie EI when ballast depth is varied.

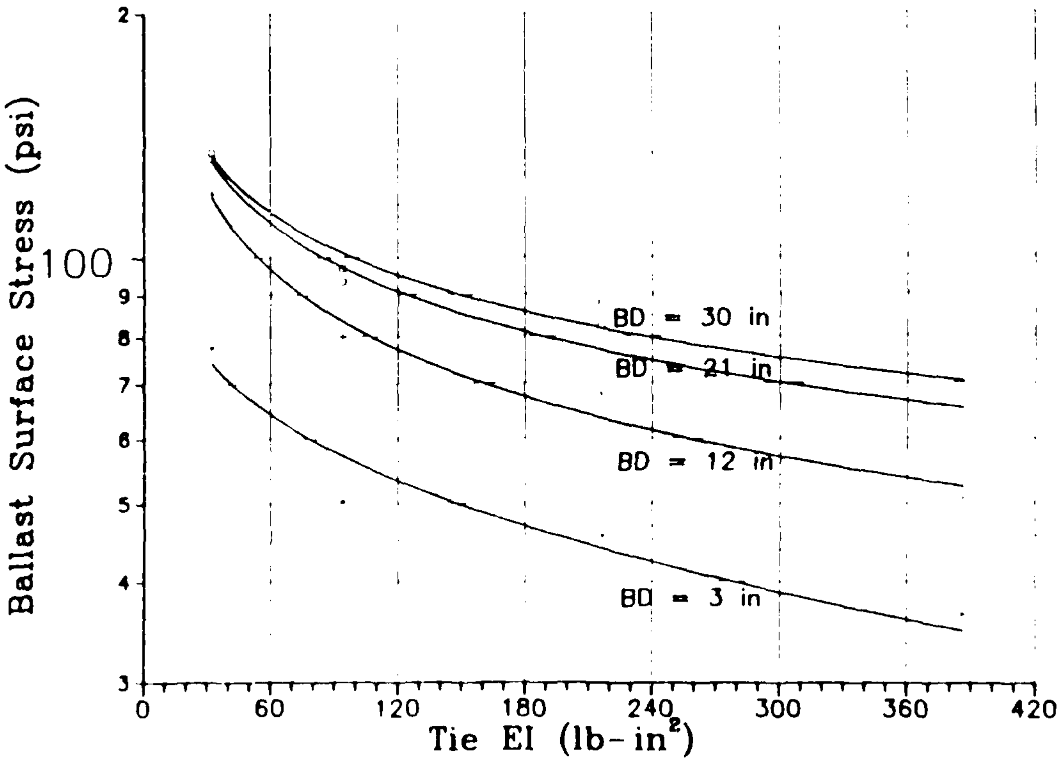

Figure 21. Log ballast surface stress vs. tie EI when ballast depth is varied.

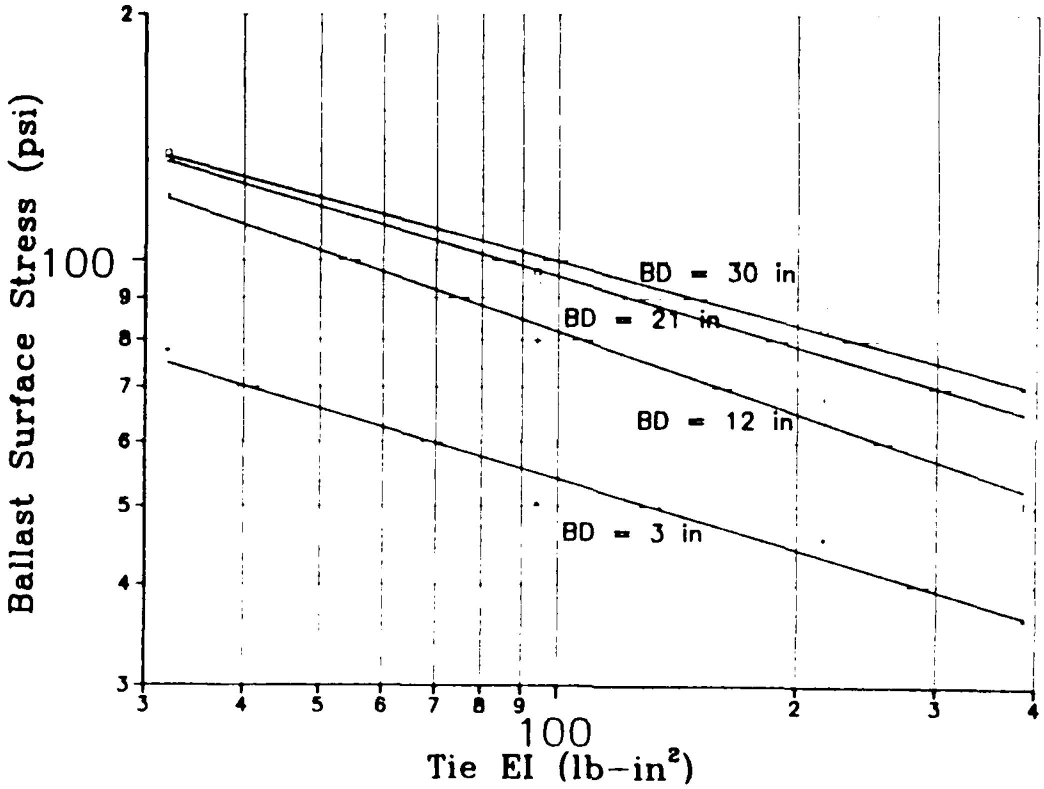

Figure 22. Log ballast surface stress vs. log tie EI when ballast depth is varied.

Significant Variables to Tie Reaction

1. Tie Spacing
2. Lg Subgrade Modulus of Elasticity
3. Lg Tie Modulus of Elasticity
4. Lg Rail Weight
5. Lg Tie Moment of Inertia
6. Tie Spacing x Lg Subgrade Modulus of Elasticity
7. Tie Spacing x Lg Rail Weight
8. Tie Spacing x Lg Tie Moment of Inertia
9. Lg Subgrade Modulus of Elasticity x Lg Tie Moment of Inertia
10. Lg Subgrade Modulus of Elasticity x Lg Tie Moment of Inertia
11. Lg Tie Modulus of Elasticity x Lg Tie Moment of Inertia

Figure 23. Significant variables from factorial analysis.

With the basic terms selected, the next step was to refine them into an equation that would closely approximate the results from the KENTRACK program. The SPSS program was used to assist in this process. The most significant single variable and multiple variable combinations selected by the factorial analysis program were chosen as input terms. After comparison with the KENTRACK data set, SPSS would then calculate coefficients for each term, along with a single constant. These terms, multiplied by the corresponding coefficients and then summed (along with the constant), formed the equation.

The equation produced in this way then became input for the CHECK program. CHECK calculated the equation for each entry (set of variable values) in the data set. The results were then compared with the corresponding values produced by KENTRACK. The output of CHECK listed all entries for which the difference between the equation and KENTRACK results were greater than ± 10 percent. The output also listed the average and maximum errors (both positive and negative), the error range, and the sum of errors. Figure 24 is sample output from this program.

It was at this point that the limits of the analysis programs were reached. Then began the long, difficult, iterative process of refining the equations until the CHECK program indicated that errors were within acceptable limits.

The analysis methods could help produce trial equations, but they were severely limited in their ability to indicate how improvements should be made (see **Obstacles to Development**). Suggestions for improvement depended on developing a sense of the analytical parameter's behavior with respect to the equation variables. This sense was acquired by examining the output from all analytical methods and observing the effect of changes made to the equation. This process required many hours of concentrated study and much trial and error.

The equation refinement process also required many iterations of the SPSS and CHECK programs. Using these programs, different terms were tried, and modified, in an attempt to improve the results. Again, the objective was to create the best equation using the fewest terms.

When the results were still unsatisfactory after many iterations, improvements were sometimes obtained by trying different forms of a variable and, as a last resort, adding more terms to the equation. These iterations continued until each equation was tuned to its final form.

Equations--Final Product

The original goal for the accuracy of the equations was to come within ± 20 percent of the KENTRACK values for every entry in the data set. This goal was met or exceeded by all equations except the one for ballast surface stress, where approximately 4 percent of the values were outside the desired range.

The rail stress equation provides the closest match to the data set. With this equation, all but 1 percent of the values were within ± 10 percent of the KENTRACK-produced values. This result significantly exceeded the original goal.

Figures 25 through 30 give the final form of each equation. The Appendix includes a computer diskette with an executable program for quick and convenient use of the track structural analysis equations. As shown in the figures, the coefficients in the equations typically run to 7 or 8 decimal places. Since these equations are only approximations, such attempted "accuracy" seems, at first, illogical.

During the refinement stage, several attempts were made to simplify the coefficients to 2 or 3 decimal places. In each case, the simplification resulted in a significant loss of fit with the data. Some of this behavior can be understood in noting that many equation variables have been expressed in log form, thus making them sensitive to small changes in the value of the coefficients. However, the apparent extreme degree of sensitivity was unexpected.

Since the equations were intended for use in a microcomputer, it appeared that leaving the coefficients in an unsimplified form would present no extra inconvenience to the user. Thus, it was decided to leave all of the decimal places in the coefficients and use the remaining time available to refine the equations to the best fit with the KENTRACK data. Perhaps during future work, a way can be found to simplify the coefficients without sacrificing accuracy of fit with the data.

RUN	TS	SUBE	BD	TIEEI	KSTR	LGKSTR	QSTR	LGQSTR	%ERROR
111	22	10000	21	32000000	15.4	1.18752	17.8	1.25093	15.72
112	22	10000	21	94000000	14.4	1.15836	16.1	1.20757	12.00
113	22	10000	21	216000000	13.3	1.12385	14.9	1.17408	12.26
114	22	10000	21	286000000	13.0	1.11394	14.5	1.16279	11.90
115	22	10000	21	386000000	12.6	1.10037	14.1	1.15072	12.29
116	22	10000	30	32000000	11.4	1.05690	12.9	1.11112	13.30
117	22	10000	30	94000000	11.0	1.04139	12.1	1.08452	10.44
118	22	10000	30	216000000	10.5	1.02119	11.6	1.06398	10.35
186	44	10000	12	32000000	50.9	1.70672	41.9	1.62196	-17.73
187	44	10000	12	94000000	43.7	1.64048	35.6	1.55188	-18.45
188	44	10000	12	[illegible]	37.6	1.57519	31.5	1.49777	-16.33
189	44	10000	12	[illegible]	35.5	1.55023	30.2	1.47952	-15.03
190	44	10000	12	386000000	33.5	1.52504	28.8	1.46002	-13.91
206	44	5000	12	32000000	37.5	1.57403	32.5	1.51245	-13.22
207	44	5000	12	94000000	32.4	1.51055	28.2	1.44967	-13.08
208	44	5000	12	216000000	28.0	1.44716	25.2	1.40119	-10.04
225	44	2750	3	386000000	53.3	1.72673	47.9	1.68069	-10.06
245	44	1500	3	386000000	41.4	1.61700	36.8	1.56614	-11.05
261	66	10000	3	32000000	221.2	2.34479	263.3	2.42047	19.04
262	66	10000	3	94000000	165.5	2.21880	191.0	2.28110	15.43
263	66	10000	3	216000000	128.5	2.10890	149.1	2.17349	16.04
264	66	10000	3	286000000	118.2	2.07262	137.1	2.13719	16.03
265	66	10000	3	386000000	108.6	2.03583	125.4	2.09841	15.50
266	66	10000	12	32000000	63.3	1.80140	52.4	1.71895	-17.29
267	66	10000	12	94000000	53.7	1.72997	44.6	1.64887	-17.03
268	66	10000	12	216000000	45.6	1.65896	39.3	1.59477	-13.74
269	66	10000	12	286000000	43.0	1.63347	37.7	1.57651	-12.29
270	66	10000	12	386000000	40.3	1.60531	36.1	1.55701	-10.52
283	66	5000	3	216000000	100.0	2.00000	110.1	2.04176	10.09
286	66	5000	12	32000000	47.3	1.67486	40.7	1.60945	-13.98
287	66	5000	12	94000000	40.4	1.60638	35.2	1.54666	-12.85
334	66	1500	21	286000000	11.3	1.05308	12.5	1.09550	10.26

SUM OF THE ABSOLUTE VALUES OF ERROR = 1256.60
DIFFERENCE OF THE POSITIVE AND NEGATIVE ERRORS = 37.49
AVERAGE POSITIVE ERROR = 5.47%   AVERAGE NEGATIVE ERROR = -5.00%
MAXIMUM POSITIVE ERROR = 19.04%   MAXIMUM NEGATIVE ERROR = -18.45%
THE NUMBER OF ERRORS OUT OF RANGE = 32

Figure 24. Sample check output for subgrade surface stress.

Variable Definition

W_r = Rail Weight (lbs)

I_r = Rail Moment of Inertia (in^4)

S_t = Tie Spacing (in)

I_t = Tie Moment of Inertia (in^4)

E_t = Tie Modulus of Elasticity (x 1,000,000 psi)

D_b = Ballast Depth (in)

E_b = Ballast Modulus of Elasticity (psi)

E_s = Subgrade Modulus of Elasticity (psi)

P_w = Wheel Load (lbs)

Equation Outputs - Track Stresses and Loading

σ_r = Rail Bending Stress (psi)

R_t = Tie Reaction (lbs)

σ_t = Tie Bending Stress (psi)

σ_b = Ballast Surface Stress (psi)

σ_s = Subgrade Stress (psi)

Figure 25. Equation variables and outputs.

$$\log(\sigma_r) = RB_1 + RB_2 + RB_3 + RB_4 + \log\left(\frac{P_w}{40,000}\right)$$

$$RB_2 = 0.29787 \times \log(I_r)$$

$$RB_3 = -0.1898195 \times \log(I_t) \times \log(E_t)$$

$$RB_4 = \left(\frac{1.59}{\log(I_r)}\right)^{[illegible]} \times D_b^{[illegible]} \times$$

$$[0.0084513 + 0.0129527 \times \log(E_s) - 0.015563 \times \log(E_b)]$$

$$RB_1 = 4.8725$$

Based on 320 runs of the KENTRACK program:

	Pos.	Neg.
Avg. Error	3.84%	-2.96%
Max. Error	12.90%	-10.00%

Error Analysis-

% Error between equation and KENTRACK	0-10	10-15	15+
Number of runs	316	4	0
% of runs	98.8	1.2	--

Figure 26. Rail bending stress equation.

$$\log(\sigma_t) = TB_1 + TB_2 + \ldots + TB_9 + \log\left(\frac{P_w}{40{,}000}\right)$$

$$TB_1 = -0.0093804565 \times S_t$$

$$TB_2 = -0.35799395 \times \log(E_s)$$

$$TB_3 = 0.81568739 \times \log\left(\frac{E_t}{1\times10^6}\right)$$

$$TB_4 = -0.46285988 \times \log(W_r)$$

$$TB_5 = -0.25393248 \times \log(I_t)$$

$$TB_6 = 0.11665384 \times S_t^{0.2} \times \log(E_s)$$

$$TB_7 = 0.0059988871 \times S_t \times \log(W_r)$$

$$TB_8 = -0.19966830 \times \log\left(\frac{E_t}{1 \times 10^6}\right) \times \log(I_t)$$

$$TB_9 = 5.0598549$$

Based on 432 runs of the KENTRACK program:

	Pos.	Neg.
Avg. Error	3.61%	-3.60%
Max. Error	13.51%	-12.40%

Error Analysis-

% Error between equation and KENTRACK	0-10	10-15	15+
Number of runs	422	10	0
% of runs	97.7	2.3	--

Figure 27. Tie bending stress equation.

$$\log(R_t) = TR_1 + TR_2 + \ldots + TR_9 + \log\left(\frac{P_w}{40,000}\right)$$

$$TR_1 = 0.68674167 \times \log(S_t)$$

$$TR_2 = 0.18602006 \times 10^{-5} \times E_s$$

$$TR_3 = 0.69065489 \times 10^{-5} \times E_r$$

$$TR_4 = 0.25330402 \times \log(D_b)$$

$$TR_5 = -0.62246249 \times 10^{-2} \times W_r$$

$$TR_6 = -0.38790634 \times 10^{-5} \times \log(S_t) \times E_b$$

$$TR_7 = -0.14092942 \times \log(S_t) \times \log(D_b)$$

$$TR_8 = 0.34244459 \times 10^{-2} \times \log(S_t) \times W_r$$

$$TR_9 = 3.3862435$$

Based on 432 runs of the KENTRACK program:

	Pos.	Neg.
Avg. Error	4.11%	-5.36%
Max. Error	19.73%	-9.98%

Error Analysis-

% Error between equation and KENTRACK	0-10	10-15	15-20	20+
Number of runs	423	7	2	0
% of runs	97.9	1.6	0.5	--

Figure 28. Tie reaction equation.

$$\sigma_b = (BS_1 + BS_2 + \ldots + BS_{17}) \times \frac{P_w}{40,000}$$

$$BS_1 = -615.80757 \times \log(S_t)$$

$$BS_2 = 57.668071 \times \log(E_s)$$

$$BS_3 = -0.0033248524 \times E_b$$

$$BS_4 = -271.74473 \times \log(W_r)$$

$$BS_5 = 192.38069 \times \log(D_b)$$

$$BS_6 = 75.066968 \times \log(E_t I_t)$$

$$BS_7 = 69.236941 \times \log(S_t) \times \log(E_s)$$

$$BS_8 = 0.0015894172 \times \log(S_t) \times E_b$$

$$BS_9 = 143.89098 \times \log(S_t) \times \log(W_r)$$

$$BS_{10} = 184.3939 \times \log(S_t) \times \log(D_b)$$

$$BS_{11} = 0.40960041 \times \left((\log(S_t))^2 \times [1889 - 201 \times \log(E_t I_t)] + \log(S_t) \times [211 \times \log(E_t I_t) - 1975] \right)$$

$$BS_{12} = 10.189882 \times \log(E_s) \times \log(E_t I_t)$$

$$BS_{13} = 0.77093644 \times [0.001228 \times \log(D_b)^2 \times E_b - 0.0005556 \times \log(D_b) \times E_b + 34]$$

$$BS_{14} = 0.39519583 \times E_b \times (E_t I_t)^{-0.3}$$

$$BS_{15} = -32.472070 \times \log(D_b) \times \log(E_t I_t)$$

$$BS_{16} = -36.020437 \times \log(S_t) \times \log(E_s) \times \log(D_b)$$

$$BS_{17} = 109.38586$$

Based on 2304 runs of the KENTRACK program:

	Pos.	Neg.
Avg. Error	8.19%	-6.61%
Max. Error	40.61%	-38.99%

Error Analysis-

% Error between equation and KENTRACK	0-10	10-20	20-25	25-30	30-35	35-40	40+
Number of runs	1715	491	55	28	7	7	1
% of runs	74.4	21.3	2.4	1.2	0.3	0.3	--

Figure 29. Ballast surface stress equation.

$$\log(\sigma_s) = SS_1 + SS_2 + \ldots + SS_8 + \log\left(\frac{P_w}{40{,}000}\right)$$

$$SS_1 = 1.3781149 \times \log(S_t)$$

$$SS_2 = 0.53861434 \times \log(E_s)$$

$$SS_3 = -0.84028146 \times \log(D_b)$$

$$SS_4 = -0.41251842 \times \log(E_t I_t)$$

$$SS_5 = -1.0693448 \times \log(S_t)^{0.6} \times \log(D_b)$$

$$SS_6 = -0.09849261 \times \log(E_s) \times \log(D_b) \times \left(\frac{\log(E_s)}{3.44}\right)^{0.8}$$

$$SS_7 = 0.26572226 \times \log(E_t I_t) \times \log(D_b)^{0.9}$$

$$SS_8 = 1.2519545$$

Based on 240 runs of the KENTRACK program:

	Pos.	Neg.
Avg. Error	5.50%	-5.19%
Max. Error	17.93%	-20.44%

Error Analysis-

% Error between equation and KENTRACK	0-10	10-15	15-20	20-25	25+
Number of runs	207	24	7	2	0
% of runs	86.3	10.0	2.9	0.8	--

Figure 30. Subgrade stress equation.

5 APPLICATION OF THE EQUATIONS

Intended Usage

The structural analysis equations were developed to serve two main purposes:

1. To provide an estimate of the suitability of existing track to handle its expected loading.

2. To permit an assessment of the effects of changes in the track--either improvements or deterioration. More specifically, the equations are intended as an indication of the following:

1. Are there weaknesses in the existing track system?

2. If no weaknesses are apparent, how much deterioration can occur before weaknesses do appear? (How much "reserve capacity" is there?)

3. If weaknesses are apparent, which track system components are deficient?

4. How serious are the deficiencies?

5. What improvements will eliminate the deficiencies?

Use of these equations should be considered a first step in checking the capability of existing track or in examining rehabilitation alternatives. These equations are not a substitute for railroad engineering expertise. They are one tool of several that should be used in a thorough examination of the track system and rehabilitation alternatives.

Example Applications

The following example illustrates a typical application of the structural analysis equations. Shown are the numbers used in the equations and the results of both the analysis and the evaluation that would accompany the results. In this example, it is as important to note the way in which the equations are used as it is to examine the numbers produced.

The DEH at Fort Example is told to start expecting regular traffic of fully loaded 140-ton flatcars at his installation. This makes him wonder about the adequacy of the track, since these cars are heavier than ones handled previously.

From inspection information and his knowledge of the track, he establishes the following data for use with the structural analysis equations:

1. Rail: 75 lb, in good condition with very little wear.

2. Ties: Most are 6 in. thick and 8 in. wide, and in fair to good condition, with average spacing of 22 in.

3. Ballast: at the top, filling in between the ties is good, clean crushed rock, but this extends down only 3 in. below the bottom of the tie, which is the factor that counts in load support.

4. Subgrade: medium-soft but acceptable; drainage is fair to good.

5. Wheel loads: the static wheel load of the loaded cars is about 31,000 lbs but a dynamic addition is needed to allow for a speed of 25 mph and various track and wheel irregularities. Total wheel load would then be about 40,000 lb.

Through his knowledge of the subject and consultation with a railroad engineer, the DEH establishes the values to be used in the structural analysis equations. These values are shown in Table 4 for Case 1 and represent the current track system.

Step 1

The values from Table 4 are substituted into the equations, and the results are shown in Table 5. Also in Table 5 are the desired limiting (or maximum) values. These limits are intended only for this situation and were established from the same source as the values in Table 4. The last column in Table 5 compares the output from the equations with the desired limiting values.

Table 4

Values Used in Example Case 1

Variable	Value	
Rail weight	75	lb
Rail moment of inertia	22.9	$in.^4$
Tie spacing	22	in.
Tie moment of inertia	144.0	$in.^4$
Tie modulus	1,000,000	lb/sq in.
Ballast depth	3	in.
Ballast modulus	35,000	lb/sq in.
Subgrade modulus	3,000	lb/sq in.
Wheel load	40,000	lb

Table 5

Results for Example Case 1

	Equation Output	Desired Limit	% of Limit
Rail bending stress	21,869 psi	26,000 psi	84
Tie reaction	19,130 lb	23,000 lb	83
Tie bending stress	1,559 psi	1,400 psi	111
Ballast surface stress	64 psi	65 psi	98
Subgrade surface stress	32 psi	18 psi	178

Table 5 shows that rail bending stress and tie reaction are within allowable limits, but tie bending stress is 11 percent too high, ballast stress is very near the limit, and subgrade stress is 78 percent too high--almost double the limit.

Thus, if the 140-ton cars are run over the existing track, the DEH should expect accelerated track settling. Since the subgrade soil and drainage are already said to be acceptable, the DEH needs to look for a way to considerably reduce the subgrade stress. Also, as the equation output indicates, it would be desirable to slightly reduce tie bending stress.

Step 2

The DEH realizes that the 3 in. of real ballast under the track is insufficient, even for the current traffic, so he decides to see what would happen if 6 in. more were added to the track (case 2). Table 6 shows the results, plus a comparison with the existing track.

The results show that adding 6 in. of ballast does substantially reduce the subgrade stress, placing it at the allowable limit. However, tie bending stress has not been affected, and ballast stress has now increased to 83 psi--30 percent over the allowable limit. (Note: this behavior is a good example of the complex interaction of the track system components.)

Step 3

Since the desired reduction in subgrade stress has been achieved by adding the 6 in. of ballast, the DEH decides to keep that change, at least temporarily, and add another improvement to learn if ballast stress can be reduced. The DEH knows that the 75-lb rail is very light by commercial standards, so he considers what replacing it with 115-lb rail (the lighter commercial rail) would do (case 3). Table 7 shows the results.

Table 6

Example Case 2 Results: Add 6 in. of Ballast

	Initial Case	**Add 6 in. Ballast**	**% of Limit**
Rail bending stress	21,869 psi	19,425 psi	75
Tie reaction	19,130 lb	20,526 lb	89
Tie bending stress	1,559 psi	1,559 psi	111
Ballast surface stress	64 psi	83 psi	130
Subgrade surface stress	32 psi	18 psi	100

Table 7

Example Case 3 Results: Add 6 in. of Ballast and 115-lb Rail

	With 6 in. Added Ballast	6 in. Added Ballast and 115 lb Rail	% of Limit
Rail bending stress	19,425 psi	13,389 psi	5[illegible]
Tie reaction	20,526 lbs	17,668 lbs	77
Tie bending stress	1,559 psi	1,353 psi	97
Ballast surface stress	83 psi	69 psi	106
Subgrade surface stress	18 psi	18 psi	100

Replacing the 75-lb rail with 115-lb rail has effectively solved the rest of the problem. Except for ballast stress, all categories are now within the desired limits, and ballast stress is only slightly high.

The DEH could stop here, deciding to add 6 in. of ballast to the track and install 115-lb rail; however, these corrections would involve considerable expense. Thus, it is desirable to find at least one possible alternative.

Step 4

On the existing track, nearly all of the ties have 6 in. by 8 in. cross sections, whereas standard commercial main line ties have 7 in. by 9 in. cross sections. Perhaps upgrading the ties would be a reasonable alternative to replacing the rail (case 4). Table 8 shows the results of this analysis compared with case 3.

Table 8

Example Case 4 Results: Upgrade Ties

	6 in. Added Ballast and 115-lb Rail	6 in. Added Ballast and 7 x 9 in. Ties	% of Limit
Rail bending stress	13,389 psi	19,425 psi	75
Tie reaction	17,668 lbs	20,526 lbs	89
Tie bending stress	1,353 psi	1,345 psi	96
Ballast surface stress	69 psi	69 psi	106
Subgrade surface stress	18 psi	17 psi	94

Table 8 shows that installing 7 in. by 9 in. ties, instead of replacing the rail, could also effectively accomplish the goal. As with the rail replacement option, ballast stress is still slightly high, but all other values are under the limit. Also, compared with the rail replacement option, the subgrade stress has been lowered slightly.

As a result of this analysis, the DEH decides to consider the tie/ballast combination in his budget. This appears to be a good, cost-effective choice since, from a structural improvement perspective, it will produce about the same result as the rail/ballast combination at much lower expense.

In this case, though, even the tie/ballast combination is more costly than the current budget will allow, so the DEH considers the following action: add the 6 in. of ballast this year and, as future budgets permit, begin replacing deteriorated ties with 7 in. by 9 in. ties. Though results are not immediate, over time this action will provide the kind of track needed to support the 140-ton flatcars properly. This plan (6 in. ballast, gradual tie upgrade) will now be a starting place for a more detailed engineering study. If the study confirms the validity of the plan, the plan will be adopted.

The example above was intended to demonstrate that, with reasonable values for the variables, and with knowledge of the relative costs of basic track work, the equations can be used to indicate feasible, cost-effective alternatives for producing a track system with the required structural capability.

6 CONCLUSIONS AND RECOMMENDATIONS

A railroad track structural analysis method has been developed to simplify assessment of a track's ability to withstand expected loads and to indicate how changes will affect the track. The method uses five equations that characterize rail bending stress, subgrade surface stress, tie bending stress, tie reaction, and ballast surface stress. These output parameters had been selected as the ones most important for track structural analysis.

The five equations can provide a simplified, yet meaningful and versatile, track structural analysis for conventional track. These equations appear valid for the full range of loads and conditions normally encountered in a track system. An example application has been presented to demonstrate their intended use and a computer diskette is provided with an executable program for convenience (see Appendix).

Although the equations were developed using a car with three-axle trucks, analysis indicates that they are equally valid for use with conventional two-axle trucks--in most cases, with no modifications to the output. However, it is recommended that for two-axle trucks with dynamic wheel loads greater than 35,000 lb on track with less than 6 in. ballast, and where the subgrade is medium soft or softer, the equation results be multiplied by 0.9.

The choice of values to use in the equations, as well as limiting values, currently remains a matter of judgment based on track inspection and/or previous records. To assist in evaluating rehabilitation alternatives, the relative costs of various track work should be obtained from local contractors or other reliable sources.

The basic track structural analysis method has been completed and validated. Future work will involve simplifying the choice of equation and limiting values as well as the analysis of results. When the method has been refined, it is recommended that it be incorporated into the RAILER system.

Metric Conversion Table

1 in.	=	2.54 cm
1 ft	=	0.305 m
1 yd	=	0.914 m
1 sq in.	=	6.45 cm^2
1 lb	=	0.453 kg
1 ton	=	907.2 kg
1 psi	=	6.895 kPa
1 mph	=	1.609 km/hr

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APPENDIX A:

COMPUTER PROGRAM FOR EASIER USE OF TRACK EQUATIONS

Track Equations 1.0

The program Track Equations is a FORTRAN implementation of the five track structural evaluation equations developed at USACERL described in this report:

1. Rail Bending stress
2. Tie Bending stress
3. Tie Reaction force
4. Ballast Surface stress
5. Subrgrade Surface stress

The following input values are from step 1 of the example from Chapter 5 (see Table 4.) Note that this program does *not* help the user to choose the field values required for input.

To run the program: (on any MS-DOS machine with 128K RAM or more)

-- insert the program disk supplied with this report into drive A:
-- type "A:" and press Enter key
-- type "trackeq" and press Enter key

The title screen, Screen 1, should now appear. Press Enter to go to the input screen, Screen 2. The user is then prompted for input values for the following nine variables:

1. Rail Weight (lb)
2. Rail Moment of Inertia (in^4)
3. Tie Spacing (in)
4. Tie Moment of Inertia (in^4)
5. Tie Modulus of Elasticity (x 1,000,000 psi)
6. Ballast Depth (in)
7. Ballast Modulus of Elasticity (x 1000 psi)
8. Subgrade Modulus of Elasticity (x 1000 psi)
9. Wheel Load (x 1000 lb)

For variables 5, 7, 8 and 9, the number the user enters is multiplied by a constant (see Screen 2.) As an example, if one enters 1.50 for the tie modulus, the program will multiply this by 1 million and use 1.5 million in its computations.

After the last entry on Screen 2, the program will show the results of the five equations (see Screen 3). The user is then prompted -

WOULD YOU LIKE TO PRINT? (Y/N)

- Entering "Y" echos the inputs and outputs to the printer
- Entering "N" prompts the question:

WOULD YOU LIKE TO INPUT NEW DATA (Y/N)

- Entering "Y" returns to Screen 2
- Entering "N" exits the program to DOS.

During the second time through Screen 2, the previous values entered are the defaults. Therefore, if the user simply presses Enter for that variable, the last entered value will be used.

TRACK EQUATIONS 1.0

TO USE PRIOR VALUES, HIT <RETURN>

Input the Rail Weight (lbs)	75.
Rail Moment of Inertia (in^4)	22.9
Tie Spacing (in.)	22.0
Tie Moment of Inertia (in^4)	144.
Tie Modulus (X 1,000,000 psi)	1.00
Ballast Depth (in.)	3.
Ballast Modulus (X 1000 psi)	35.
Subgrade Modulus (X 1000 psi)	3.0
Wheel Load (X 1000 lbs)	40.0

After the last value is entered, the following screen will appear displaying both the inputs, for verification, and the outputs:

TRACK EQUATIONS 1.0

	INPUTS	OUTPUTS	
RAIL WEIGHT	75.0 lbs.		
RAIL MOM. OF INERTIA	22.9 in^4		
TIE SPACING	22.0 in.	RAIL BENDING STRESS	21868.56 psi
TIE MOM. OF INERTIA	144.0 in^4	TIE REACTION	19130.11 lbs
TIE MODULUS	1000000.0 lb/in^2	TIE BENDING STRESS	1559.04 psi
BALLAST DEPTH	3.0 in.	BALLAST SURFACE STRESS	64.46 psi
BALLAST MODULUS	35000.0 lb/in^2	SUBGRADE STRESS	32.48 psi
SUBGRADE MODULUS	3000.0 lb/in^2		
WHEEL LOAD	40000.0 lbs.		

WOULD YOU LIKE TO PRINT?(Y/N)---------------> Y

If you would like to print inputs and outputs in the same form as that which is on the monitor, then the correct response to the query is Y [Enter]. Note that the default to this question is always the affirmative. Only an N will cause the program to bypass the print mode; any other input (even [Enter]) will enable the printer.

You will then be prompted WOULD YOU LIKE TO INPUT NEW DATA? (Y/N). If you would care to try some new values, type Y [Enter]. Again, the default is affirmative. Typing N will cause the program to end.

When entering a new set of data, you can use the same values as those used previously by simply hitting the enter key for that input. This procedure, of course, will not work the first time the program is started.